Math 152/172 **WEEK in REVIEW 8** Spring 2016

Sections 10.3, 10.4.

10.3 : The Integral and Comparison Tests; Estimating Sums

- THE TEST FOR DIVERGENCE: If $\lim_{n\to\infty} a_n$ does not exist or if $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum a_n$ is divergent.
- THE INTEGRAL TEST: Let $\sum a_n$ be a **positive** series. If f is a continuous and decreasing function on $[a,\infty)$ such that $a_n = f(n)$ for all $n \geq a$ then $\sum a_n$ and $\int_a^\infty f(x)\,dx$ both converge or both diverge.
- THE COMPARISON TEST: Suppose that $\sum a_n$ and $\sum b_n$ are series with **nonnegative** terms and $a_n \leq b_n$ for all n.
  1. If $\sum b_n$ is convergent then $\sum a_n$ is also convergent.
  2. If $\sum a_n$ is divergent then $\sum b_n$ is also divergent.
- LIMIT COMPARISON TEST: Suppose that $\sum a_n$ and $\sum b_n$ are series with **positive** terms . If

$$\lim_{n\to\infty} \frac{a_n}{b_n} = c$$

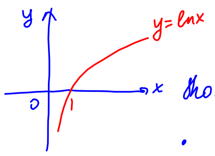
  where c is a finite number and $c > 0$, then either both series converge or both diverge.
- The p-series, $\boxed{\sum_{n=1}^{\infty} \frac{1}{n^p}, \text{ converges if } p > 1 \text{ and diverges if } p \leq 1.}$
- REMAINDER ESTIMATE FOR THE INTEGRAL TEST: If $\sum a_n$ converges by the Integral Test and $R_n = s - s_n$, then

$\left(\sum_{n=1}^{\infty} a_n \approx s_n\right)$

$$\boxed{\int_{n+1}^{\infty} f(x)\,dx \leq R_n \leq \int_n^{\infty} f(x)\,dx}$$

Examples.

1. Determine if the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^4}$ is convergent or divergent.



Integral test. $f(x) = \dfrac{1}{x(\ln x)^4}$

Show that $f(x)$ is positive, continuous and decreasing on $[2,\infty)$.

- if $x \geq 2$, then $\ln x > 0$ and $\dfrac{1}{x(\ln x)^4} > 0$
- $f(x) = \dfrac{1}{x(\ln x)^4}$ is discontinuous when $x(\ln x)^4 = 0$

$x = 0$ or $\ln x = 0$

$x = 1$

$x=0$ and $x=1$ are not in $[2,\infty)$

$\dfrac{1}{x(\ln x)^4}$ is continuous on $[2,\infty)$

- $f'(x) = -\dfrac{1}{[x(\ln x)^4]^2}\left(x(\ln x)^4\right)' = -\dfrac{(\ln x)^4 + 4x(\ln x)^3\frac{1}{x}}{x^2(\ln x)^8} = -\dfrac{(\ln x)^3[\ln x + 4]}{x^2(\ln x)^8}$

$= -\dfrac{\ln x + 4}{x^2(\ln x)^5} < 0$

$f(x) = \dfrac{1}{x(\ln x)^4}$ is decreasing on $[2,\infty)$.

$$\int_2^{\infty} \frac{dx}{x(\ln x)^4} = \lim_{t\to\infty} \int_2^t \frac{dx}{x(\ln x)^4} = \left| \begin{array}{l} u = \ln x \\ du = \frac{1}{x}dx \\ 2 \to \ln 2 \\ t \to \ln t \end{array} \right| = \lim_{t\to\infty} \int_{\ln 2}^{\ln t} \frac{du}{u^4} = \lim_{t\to\infty} \left.\frac{u^{-3}}{-3}\right]_{\ln 2}^{\ln t}$$

$$= -\frac{1}{3}\lim_{t\to\infty}\left[(\ln t)^{-3} - (\ln 2)^{-3}\right]$$

$$= -\frac{1}{3}\left[\lim_{t\to\infty} \overbrace{\frac{1}{(\ln t)^3}}^{0}\right] + \frac{1}{3(\ln 2)^3} = \frac{1}{3(\ln 2)^3} \text{ converges.}$$

$$\boxed{\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^4} \text{ converges by the Integral Test.}}$$

2. Find the values of p for which the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ is divergent.

$f(x) = \frac{1}{x(\ln x)^p}$ is positive, continuous on $[2, \infty)$

$$f'(x) = -\frac{1}{[x(\ln x)^p]^2}\left(x(\ln x)^p\right)' = -\frac{(\ln x)^p + \cancel{x}p(\ln x)^{p-1}\frac{1}{\cancel{x}}}{x^2(\ln x)^{2p}}$$

$$= -\frac{(\ln x)^p + p(\ln x)^{p-1}}{x^2(\ln x)^{2p}} = -\frac{(\ln x)^{p-1}[\ln x + p]}{x^2(\ln x)^{2p}}$$

$$= -\frac{\ln x + p}{x^2(\ln x)^{2p-(p-1)}} < 0 \text{ on } [2, \infty)$$

$f(x)$ is decreasing on $[2, \infty)$.

$$\int_2^{\infty} \frac{dx}{x(\ln x)^p} = \lim_{t\to\infty} \int_2^t \frac{dx}{x(\ln x)^p} \quad \left|\begin{array}{l} u = \ln x \\ du = \frac{dx}{x} \\ 2 \to \ln 2 \\ t \to \ln t \end{array}\right| = \lim_{t\to\infty} \int_{\ln 2}^{\ln t} \frac{du}{u^p} = \lim_{t\to\infty} \left.\frac{u^{-p+1}}{-p+1}\right]_{\ln 2}^{\ln t}$$

$$= \frac{1}{-p+1}\Big[\underbrace{\lim_{t\to\infty}(\ln t)^{-p+1}}_{\text{zero, when } -p+1<0} - (\ln 2)^{-p+1}\Big] = \begin{cases} \text{converges, if } -p+1<0 \\ \text{diverges, if } -p+1>0 \end{cases}$$

if $p=1$, then $\int_2^{\infty} \frac{dx}{x\ln x} = \lim_{t\to\infty} \int_2^t \frac{dx}{x\ln x} \quad \left|\begin{array}{l} u = \ln x \\ du = \frac{dx}{x} \end{array}\right|$

$$= \lim_{t\to\infty} \int_{\ln 2}^{\ln t} \frac{du}{u} = \lim_{t\to\infty} \ln|u| \Big]_{\ln 2}^{\ln t} = \Big[\lim_{t\to\infty} \overset{\infty}{\cancel{\ln|\ln t|}} - \ln|\ln 2|\Big] = \infty$$

divergent.

$\boxed{\int_2^{\infty} \frac{dx}{x(\ln x)^p} \text{ is divergent, if } p \le 1}$ $\boxed{\text{By the Integral test, } \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p} \text{ is divergent, if } p \le 1}$

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent, if $p>1$.

3. Determine if the following series is convergent or divergent:

(a) $\sum_{n=1}^{\infty} \frac{2012}{\sqrt[7]{n^5}\sqrt[3]{8n}} = \sum_{n=1}^{\infty} \frac{2012}{2\,n^{5/7+1/3}} = \sum_{n=1}^{\infty} \frac{2012}{2\,n^{\frac{15+7}{21}}} = \sum_{n=1}^{\infty} \frac{2012}{2\,n^{\frac{22}{21}}}$ $\left(p=\frac{22}{21}>1\right)$

$\boxed{\text{convergent}}$

(b) $\sum_{n=1}^{\infty} \frac{n^2+12}{\sqrt{n^6+6}} \sim \sum_{n=1}^{\infty} \frac{n^2}{\sqrt{n^6}} = \sum_{n=1}^{\infty} \frac{n^2}{n^3} = \underline{\sum_{n=1}^{\infty} \frac{1}{n}}$ - divergent.

$\lim_{n\to\infty} \frac{\frac{n^2+12}{\sqrt{n^6+6}}}{n} = \lim_{n\to\infty} \frac{n(n^2+12)}{\sqrt{n^6+6}} = 1 > 0$, $\neq \infty$.

$\sum_{n=1}^{\infty} \frac{n^2+12}{\sqrt{n^6+6}}$ diverget by the limit comparison Test.

(c) $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^7}\right)$

$\frac{1}{n^7} \to 0$, if $n\to\infty$. near zero $\sin x \approx x$

$\sin\frac{1}{n^7} \approx \frac{1}{n^7}$. Compare with $\sum_{n=1}^{\infty} \frac{1}{n^7}$ (convergent $p=7>1$)

$\lim_{n\to\infty} \frac{\sin\frac{1}{n^7}}{\frac{1}{n^7}} \underset{x\to 0}{\overset{x=\frac{1}{n^7}}{=}} \lim_{x\to 0} \frac{\sin x}{x} = 1 > 0$

$\boxed{\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^7}\right) \text{ is convergent by Limit Comparison Test}}$

(d) $\sum_{n=1}^{\infty} \frac{5n^5+e^{-5n}}{6n^6-e^{-6n}}$ $\boxed{\lim_{x\to\infty} e^{-x} = 0.}$

$\sim \sum_{n=1}^{\infty} \frac{5n^5}{6n^6} = \sum_{n=1}^{\infty} \frac{5}{6n}$, and $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.

should use the comparison Test.

$\frac{5n^5+e^{-5n}}{6n^6-e^{-6n}} > \frac{5n^5}{6n^6-e^{-6n}} > \frac{5n^5}{6n^6} = \frac{5}{6n}$

$\frac{1}{2-1} > \frac{1}{2}$, $\sum_{n=1}^{\infty} \frac{5n^5+e^{-5n}}{6n^6-e^{-6n}}$ compare with $\sum_{n=1}^{\infty} \frac{5}{6n}$

$\boxed{\text{By Comparison Test, } \sum_{n=1}^{\infty} \frac{5n^5+e^{-5n}}{6n^6-e^{-6n}} \text{ is divergent}}$

4. Find the values of p for which the series $\sum_{n=1}^{\infty} \frac{1}{(n+1)n^p}$ is convergent.

$$\sum_{n=1}^{\infty} \underbrace{\frac{1}{(n+1)n^p}}_{a_n} \sim \sum_{n=1}^{\infty} \underbrace{\frac{1}{n^{p+1}}}_{b_n} \quad \text{converges if } p+1>1 \text{ or } p>0$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{1}{(n+1)n^p}}{\frac{1}{n^{p+1}}} = \lim_{n\to\infty} \frac{n^{p+1}}{(n+1)n^p} = 1 > 0.$$

$\sum_{n=1}^{\infty} \frac{1}{(n+1)n^p}$ and $\sum_{n=1}^{\infty} \frac{1}{n^{p+1}}$ either both converge or both diverge.

$\sum_{n=1}^{\infty} \frac{1}{n^{p+1}}$ converges, if $p>0$, therefore $\boxed{\sum_{n=1}^{\infty} \frac{1}{(n+1)n^p} \text{ is convergent when } p>0}$

$$\int_{n+1}^{\infty} f(x)\,dx \le R_n \le \int_{n}^{\infty} f(x)\,dx$$

5. (a) If $\sum_{n=1}^{1000} \frac{1}{n^6}$ is used to approximate $\sum_{n=1}^{\infty} \frac{1}{n^6}$, find an upper bound on the error using the Integral Test.

$$\sum_{n=1}^{\infty} \frac{1}{n^6} \approx S_{1000}, \quad f(n) = \frac{1}{n^6}, \quad f(x) = \frac{1}{x^6}$$

$$\int_{1000+1}^{\infty} \frac{dx}{x^6} \le R_{1000} \le \int_{1000}^{\infty} \frac{1}{x^6}\,dx$$

$$\frac{1}{5(1001)^5} \le \boxed{R_{1000} \le \frac{1}{5(1000)^5}}$$

$$\int_a^{\infty} \frac{1}{x^6}\,dx = \lim_{t\to\infty} \int_a^t x^{-6}\,dx = \lim_{t\to\infty} \left.\frac{x^{-5}}{-5}\right]_a^t = \lim_{t\to\infty}\left[\cancel{\frac{t^{-5}}{-5}}\right] - \frac{a^{-5}}{-5}$$

$$0$$

$$= \frac{a^{-5}}{5} = \frac{1}{5a^5}$$

(b) Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^6}$ correct to 11 decimal places.

$$R_n < 10^{-11}$$

$$R_n < \int_n^{\infty} f(x)\,dx = \frac{1}{5n^5}$$

$$\frac{1}{5n^5} < 10^{-11} \quad \text{or} \quad 5n^5 > 10^{11}$$

$$n^5 > \frac{10^{11}}{5}$$

$$n > \sqrt[5]{\frac{10^{11}}{5}} = \underbrace{10^{\frac{11}{5}}}_{158.5} \cdot \underbrace{\frac{1}{\sqrt[5]{5}}}_{0.72} \approx 114.87$$

$$n = 115.$$

$$\boxed{\sum_{n=1}^{\infty} \frac{1}{n^6} \approx \frac{1}{1} + \frac{1}{2^6} + \frac{1}{3^6} + \ldots + \frac{1}{(115)^6}}$$

6. Given the series $\sum_{n=1}^{\infty} n^3 e^{-n^4}$.

(a) Show that the series converges.

$f(x) = x^3 e^{-x^4} > 0$ on $[1,\infty)$

continuous on $[1,\infty)$

$f'(x) = 3x^2 e^{-x^4} + x^3 e^{-x^4}(-4x^3)$ positive on $[1,\infty)$

$= 3x^2 e^{-x^4} - 4x^6 e^{-x^4} = \underbrace{x^2}[3-4x^4]\underbrace{e^{-x^4}}_{\text{positive on } [1,\infty)}$

$3-4x^4<0$ on $[1,\infty)$

thus, $f'(x)<0$ on $[1,\infty)$.

$f(x)$ is decreasing on $[1,\infty)$.

$\int_1^\infty x^3 e^{-x^4}dx = \lim_{t\to\infty}\int_1^t x^3 e^{-x^4}dx = \left| \begin{array}{l} u=-x^4 \\ du=-4x^3dx \\ x^3dx = -\frac{du}{4} \\ 1 \to -(1)^4=-1 \\ t \to -t^4 \end{array} \right| = \lim_{t\to\infty}\left[-\frac{1}{4}\int_{-1}^{-t^4} e^u du\right] = \lim_{t\to\infty}\left(-\frac{1}{4}\cancel{e^{-t^4}}^{0} + \frac{1}{4}e^{-1}\right)$

$= \frac{1}{4e}$ convergent.

$\boxed{\sum_{n=1}^{\infty} n^3 e^{-n^4} \text{ converges by the Integral Test}}$

(b) Find an upper bound for the error approximating this series by its 5th partial sum s_5.

$R_5 \le \int_5^\infty f(x)dx$

$R_5 \le \int_5^\infty x^3 e^{-x^4}dx = \lim_{t\to\infty}\left[-\frac{1}{4}\int_{-5^4}^{-t^4} e^u du = \right] = \lim_{t\to\infty} \cancel{\frac{e^{-t^4}}{-4}}^{0} - \frac{e^{-5^4}}{-4}$

$\boxed{R_5 \le \frac{1}{4e^{625}}}$

10.4 : Other Convergence Tests

- ALTERNATING SERIES TEST: If $b_n > 0$, $\lim_{n\to\infty} b_n = 0$ and the sequence $\{b_n\}$ is decreasing then the series $\sum(-1)^n b_n$ is convergent.
- RATIO TEST: For a series $\sum a_n$ with nonzero terms define $L = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$.
  1. If $L < 1$ then the series is absolutely convergent (which implies the series is convergent.)
  2. If $L > 1$ then the series is divergent.
  3. If $L = 1$ then the series may be divergent, conditionally convergent or absolutely convergent (test fails).
- **The Alternating Series Theorem.** If $\sum_{n=1}^{\infty}(-1)^n b_n$ is a convergent alternating series and you used a partial sum s_n to approximate the sum s (i.e. $s \approx s_n$) then $|R_n| \le b_{n+1}$.

Examples

conditionally converges

7. Determine whether the following series converges absolutely, converges but not absolutely, or diverges.

   (a) $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^p}$, where p is a real parameter. , $b_n = \frac{1}{n^p}$

$\lim_{n\to\infty}\frac{1}{n^p} = \begin{cases} 0, & \text{if } p \ge 0 \\ \infty, & \text{if } p < 0 \end{cases} \Rightarrow \sum_{n=1}^{\infty}\frac{(-1)^n}{n^p}$ diverges, if $p<0$

$b_n = \frac{1}{n^p}$, $p>0$. (1) $\lim_{n\to\infty}\frac{1}{n^p} = 0$.

(2) $\frac{1}{n^p}$ is decreasing.

By the alternating series Test, $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^p}$ is convergent, if $p>0$

Test for absolutely convergence.

$\sum_{n=1}^{\infty}\left|\frac{(-1)^n}{n^p}\right| = \sum_{n=1}^{\infty}\frac{1}{n^p}$ — converges, if $p>1$

$$\boxed{\sum_{n=1}^{\infty}\frac{(-1)^n}{n^p}\ \begin{cases}\text{diverges, if } p<0 \\ \text{is conditionally convergent, if } 0<p\le 1 \\ \text{is absolutely convergent, if } p>1\end{cases}}$$

(b) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n\sqrt[4]{\ln n}}$

alternating series test for $b_n = \frac{1}{n\sqrt[4]{\ln n}}$

- $\lim_{n\to\infty} b_n = \lim_{n\to} \frac{1}{n\sqrt[4]{\ln n}} = 0.$
- $f(x) = \frac{1}{x\sqrt[4]{\ln x}} \Rightarrow f'(x) = -\frac{\sqrt[4]{\ln x} + \frac{1}{4}x(\ln x)^{-3/4}\cdot\frac{1}{x}}{x^2\sqrt{\ln x}} < 0$ on $[2,\infty)$.

by the alterating series Test, $\boxed{\sum_{n=2}^{\infty}\frac{(-1)^n}{n\sqrt[4]{\ln n}} \text{ is conditionally convergent.}}$

$\sum_{n=2}^{\infty}\left|\frac{(-1)^n}{n\sqrt[4]{\ln n}}\right| = \sum_{n=2}^{\infty}\frac{1}{n\sqrt[4]{\ln n}}$ – divergent by Example 2, $p = 1/4 < 1$.

$(n+1)! = (n+1)(n)(n-1)\cdot\ldots\cdot(2)(1).$

(c) $\sum_{n=1}^{\infty} \frac{(-9)^n}{(n+1)!}$

Use the Ratio Test.

$a_n = \frac{(-9)^n}{(n+1)!}, \quad a_{n+1} = \frac{(-9)^{n+1}}{((n+1)+1)!} = \frac{(-9)^{n+1}}{(n+2)!}$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\frac{(-9)^{n+1}}{(n+2)!}}{\frac{(-9)^n}{(n+1)!}}\right| = \lim_{n\to\infty}\left|\frac{\cancel{(-9)^{n+1}}\,(n+1)!}{(n+2)!\,\cancel{(-9)^n}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{-9\,\cancel{(n+1)(n)(n-1)\cdot\ldots\cdot 2\cdot 1}}{(n+2)\cancel{(n+1)(n)(n-1)\cdot\ldots\cdot 2\cdot 1}}\right| = \lim_{n\to\infty}\left|-\frac{9}{n+2}\right| = 0 < 1$$

$\boxed{\text{absolutely convergent by the Ratio Test}}$

(d) $\sum_{n=5}^{\infty} \frac{(-1)^{n-1}7^{n-1}}{4^n} = \sum_{n=5}^{\infty}\frac{(-7)^{n-1}}{4^n} = \sum_{n=5}^{\infty}\frac{(-7)(-7)^n}{4^n} = \sum_{n=5}^{\infty}(-7)\left(-\frac{7}{4}\right)^n$

geometric series $r = -\frac{7}{4} < -1$

$\boxed{\text{diverges}}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{n\cos(n\pi)}{n^2+n+1}$

$\cos \pi n = \begin{cases} 1, & \text{if } n \text{ is even} \\ -1, & \text{if } n \text{ is odd} \end{cases} \Rightarrow \cos \pi n = (-1)^n$

$\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+n+1}$ alternating series, $b_n = \dfrac{n}{n^2+n+1}$

So the alternating series Test.

- $\displaystyle\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{n}{n^2+n+1} = 0$
- $\displaystyle\left(\frac{x}{x^2+x+1}\right)' = \frac{x^2+x+1-x(2x+1)}{(x^2+x+1)^2} = \frac{x^2+x+1-2x^2-x}{(x^2+x+1)^2} = \frac{1-x^2}{(x^2+x+1)^2} \le 0$ on $[1,\infty)$

$\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+n+1}$ is convergent.

$\displaystyle\sum_{n=1}^{\infty} \left|\frac{(-1)^n n}{n^2+n+1}\right| = \sum_{n=1}^{\infty} \underbrace{\frac{n}{n^2+n+1}}_{a_n} \sim \sum_{n=1}^{\infty} \underbrace{\frac{1}{n}}_{b_n}$ - divergent

Limit Comparison Test: $\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{n}{n^2+n+1}}{\frac{1}{n}} = 1 > 0.$

$\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+n+1}$ and $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ both diverge.

$\boxed{\displaystyle\sum_{n=1}^{\infty} \frac{n\cos\pi n}{n^2+n+1} \text{ is conditionally convergent}}$

8. Which of the following statements is TRUE?

(a) If $a_n > 0$ for $n \geq 1$ and $\sum_{n=1}^{\infty}(-1)^n a_n$ converges then $\sum_{n=1}^{\infty} a_n$ converges. not true.

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ is convergent, but $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.

(b) If $a_n > 0$ for $n \geq 1$ and $\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty}(-1)^n a_n$ converges. $\boxed{\text{TRUE}}$

if $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} (-1)^n a_n$ is absolutely convergent.
If a series is absolutely convergent, then it is convergent.

(c) If $\lim_{n\to\infty} a_n = 0$ then $\sum_{n=1}^{\infty}(-1)^n a_n$ converges. $\boxed{\text{FALSE}}$

(d) If $a_n > 0$ for $n \geq 1$ and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{e}{2}$ then $\sum_{n=1}^{\infty} a_n$ converges. $\boxed{\text{FALSE}}$

$\frac{e}{2} \approx \frac{2.7}{2} > 1$ $\quad \sum_{n=1}^{\infty} a_n$ diverges by the Ratio Test.

9. Given the series $\sum_{n=1}^{\infty}(-1)^{n+1}n^3e^{-n^4}$.

(a) Show that the series converges.

Have already shown that $\sum_{n=1}^{\infty} n^3 e^{-n^4}$ is convergent (Example 6).
Therefore, $\sum_{n=1}^{\infty} (-1)^{n+1} n^3 e^{-n^4}$ is absolutely convergent.

(b) Find an upper bound for the error approximating this series by its $\boxed{\text{5th}}$ partial sum s_5.

$|R_n| \le b_{n+1}$, $b_n = n^3 e^{-n^4}$

$|R_5| \le b_6 = 6^3 e^{-6^4}$

$$\boxed{R_5 \le \frac{6^3}{e^{6^4}}}$$