Review for the Final

1. Find the average value of the function $f(x) = \cos(ax + \pi/4)$ on the interval $\left[0, \dfrac{\pi}{a}\right]$, where a is a positive real parameter.

2. Evaluate the integral $\displaystyle\int_0^1 xe^x \, dx$

3. Evaluate the integral $\displaystyle\int_0^1 \frac{4}{(3x+1)(x-1)} \, dx$

4. The region $D = \{(x, y) : 0 \le x \le 1, 0 \le y \le x\}$ is rotated about the horizontal line $y = -2$. Find the generated volume.

5. Evaluate the integral $\displaystyle\int_{-8}^0 \frac{3x}{\sqrt{x+9}} \, dx$

6. When a spring of natural length 7m is extended to 8m, the force required to hold it in position is 20N. Find the work done (in Joules) when the spring is extended from 8m to 9m.

7. Find the area bounded by the curves $y = 3x$ and $y = x^2 + 2$ from $x = 0$ to $x = 3$.

8. Evaluate the integral $\displaystyle\int_0^{\pi/2} \sin^4 x \cos^3 x \, dx$.

9. Find the integral $\displaystyle\int \frac{1}{\sqrt{4x - x^2 - 3}} \, dx$

10. The region bounded by the curves $y = x^3$ and $y = 9x$ is rotated about the x-axis. Find the volume generated.

11. The region bounded by the lines $y = 0$, $y = -2x + 6$ and $y = 4x$ is rotated about the x-axis.Set up, but don't evaluate, integrals which give the volume generated using
    (a) the washer/disk method
    (b) the cylindrical shells method.

12. A tank was constructed by rotating about the y-axis the part of the parabola $y = x^2$ such that depth of the tank is $4ft$. The tank is then filled with a liquid solution weighing $60lb/ft^3$. Find the work done in pumping out the tank.

13. Which of these integrals represents the length of the curve $y = x^4$ from $x = 0$ to $x = 1$?
    (a) $\displaystyle\int_0^1 \sqrt{1 + x^8} \, dx$
    (b) $\displaystyle\int_0^1 \sqrt{1 + 4x^3} \, dx$
    (c) $\displaystyle\int_0^1 \sqrt{1 + 16x^6} \, dx$
    (d) $\displaystyle 2\pi \int_0^1 x^4 \sqrt{1 + 16x^6} \, dx$
    (e) $\displaystyle\int_0^1 \sqrt{1 + x^4} \, dx$

14. By comparing the functions $\dfrac{1}{1+x^5}$ and $\dfrac{1}{x^5}$ what conclusion can be drawn about $\displaystyle\int_1^\infty \frac{1}{1+x^5} \, dx$?
    (a) Its value is 1.
    (b) Its value is 1/2.

(c) It diverges.

(d) It converges.

(e) No conclusion is possible.

15. Does the integral $\displaystyle\int_0^1 \frac{1+x}{\sqrt{x}}\,\mathrm{d}x$ diverge?

16. The curve $y = e^{-3x}$ from $x = -1$ to $x = 0$ is rotated about the y-axis. Set up, but don't evaluate, integral which gives the generated surface area.

17. Given a positive series with general term a_n.

(a) **TRUE** **FALSE** If $\displaystyle\lim_{n\to\infty} a_n = 0$ then the series converges.

(b) **TRUE** **FALSE** If $a_n \geq \dfrac{1}{n^4}$ then the series converges.

(c) **TRUE** **FALSE** If $\displaystyle\lim_{n\to\infty} \frac{a_n+1}{a_n} = 1$ then the series converges.

(d) **TRUE** **FALSE** If $a_n \leq \dfrac{1}{\sqrt{n}}$ then the series diverges.

18. Find the interval of convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^n}{\sqrt[5]{n^2}}$.

19. Compute $\displaystyle\lim_{x\to 0} \frac{x^4 - \sin x^4}{1-\cos(x^6)}$.

20. The series $\displaystyle\sum_{n=2017}^{\infty} \frac{(n!)^6}{((3n)!)^6}$

(a) Diverges by the Integral Test

(b) Diverges by the Comparison Test

(c) Diverges by the Ratio Test

(d) Converges by the Ratio Test

(e) Diverges because $\displaystyle\lim_{n\to\infty} a_n = 0$.

21. Find the area bounded by the curves $y = 3x$ and $y = \sqrt{x}$.

22. A trigonometric substitution converts the integral $\displaystyle\int \sqrt{x^2+20x+75}\,\mathrm{d}x$ to

(a) $5\displaystyle\int \tan^3\theta\,\mathrm{d}\theta$

(b) $25\displaystyle\int \tan^2\theta\sec\theta\,\mathrm{d}\theta$

(c) $25\displaystyle\int \sin^3\theta\,\mathrm{d}\theta$

(d) $5\displaystyle\int \sin^2\theta\cos\theta\,\mathrm{d}\theta$

(e) $5\displaystyle\int \tan\theta\sec^2\theta\,\mathrm{d}\theta$

23. Find the average value of the function $f(x) = \sin^3 x$ on the interval $\left[0, \dfrac{\pi}{2}\right]$.

24. Evaluate the integral $\displaystyle\int x^3 \sin 3x\,\mathrm{d}x$.

25. The improper integral $\displaystyle\int_5^\infty \frac{5+\sin x}{x^{10}}\,\mathrm{d}x$

   (a) converges to the value $1/50$.
   (b) converges, because the integrand oscillates.
   (c) diverges to ∞
   (d) converges by comparison with $\displaystyle\int_5^\infty \frac{6}{x^{10}}\,\mathrm{d}x$
   (e) diverges but doesn't approach ∞, because the integrand oscillates.

26. Set up, but don't evaluate, integral which gives the arc length of the curve

$$x = 2017 + \cos(2t), \quad y = t - \sin(2t), \quad 0 \le t \le \pi/2.$$

   Circle the correct answer:

   (a) $\displaystyle\int_0^{\pi/2} \sqrt{2018 + t^2 + 2\cos(2t) - 2\sin(2t)}\,\mathrm{d}t$
   (b) $\displaystyle\int_0^{\pi/2} \sqrt{2017 + t^2 + 2\cos(2t) - 2\sin(2t)}\,\mathrm{d}t$
   (c) $\displaystyle\int_0^{\pi/2} \sqrt{2017 - 4\cos(2t) + 4\sin(2t)}\,\mathrm{d}t$
   (d) $\displaystyle\int_0^{\pi/2} \sqrt{5 - 4\cos(2t)}\,\mathrm{d}t$
   (e) $\displaystyle\int_0^{\pi/2} \sqrt{6 - 4\cos(2t)}\,\mathrm{d}t$

27. Determine whether the integral $\displaystyle\int_1^\infty \frac{1}{(x-3)^4}\,\mathrm{d}x$ is divergent or convergent.

28. Find $\displaystyle\int \frac{x^2+1}{x^3+2x^2+x}\,\mathrm{d}x$

29. Describe the surface having the equation $x^2 + y^2 + z^2 - 10x + 2z + 1 = 0$.

30. Compute $\displaystyle\int_{-1}^1 \frac{1}{x^6}\,\mathrm{d}x$

31. Which of the following series are convergent?

   (a) $\displaystyle\sum_{n=1}^\infty \frac{2017^n}{n!}$
   (b) $\displaystyle\sum_{n=1}^\infty \frac{2017^n}{n + 2018^n}$

32. Compute $\displaystyle\sum_{n=0}^\infty \frac{2017^{n-1}}{2016^n}$.

33. The series $\displaystyle\sum_{k=1}^\infty \frac{(-1)^k}{\sqrt[4]{k}}$ is

   (a) divergent to ∞
   (b) divergent to $-\infty$
   (c) divergent but not to $\pm\infty$
   (d) absolutely convergent

(e) conditionally convergent

34. Find the value(s) of x such that the vectors $\langle x, -1, 1\rangle$ and $\left\langle 1, -x^2, x^3\right\rangle$ are orthogonal.

35. Find the Taylor series for $f(x) = x^3 + x^2 + 3$ about $x = 3$.

36. Find a power series centered at $x = 0$ for the function $f(x) = \frac{x}{1-8x^3}$, and determine the radius of convergence.

37. Find the angle between the vectors $\langle 1, 2, 1\rangle$ and $\langle 3, 3, 0\rangle$.

38. Evaluate the integral $\displaystyle\int_0^{1/3} \frac{1}{1+x^7}\,\mathrm{d}x$ as infinite series.

39. Find the third degree Taylor polynomial for $f(x) = \ln x$ about $x = 5$.

40. Consider the points $A(0, 1, 4)$, $B(2, 1, 3)$, and $C(1, -1, 0)$.

    (a) Find a unit vector orthogonal to the plane determined by the given points.

    (b) Find the area of the triangle with vertices A,B, and C.

41. Which of the following statements is most accurately describes the convergence or divergence of the improper integral $\displaystyle\int_1^{\infty} \frac{x}{\sqrt{x^7+77}}\,\mathrm{d}x$?

    (a) The integral converges because $\dfrac{x}{\sqrt{x^7+77}} < \dfrac{1}{x^{7/2}}$ and the integral $\displaystyle\int_1^{\infty} \frac{1}{x^{7/2}}\,\mathrm{d}x$ converges.

    (b) The integral converges because $\dfrac{x}{\sqrt{x^7+77}} < \dfrac{1}{x^{6}}$ and the integral $\displaystyle\int_1^{\infty} \frac{1}{x^{6}}\,\mathrm{d}x$ converges.

    (c) The integral converges because $\dfrac{x}{\sqrt{x^7+77}} < \dfrac{1}{x^{5/2}}$ and the integral $\displaystyle\int_1^{\infty} \frac{1}{x^{5/2}}\,\mathrm{d}x$ converges.

    (d) The integral diverges because $\dfrac{x}{\sqrt{x^7+77}} \geq \dfrac{1}{x^{5/2}}$ and the integral $\displaystyle\int_1^{\infty} \frac{1}{x^{5/2}}\,\mathrm{d}x = \infty$.

    (e) The integral diverges because $\dfrac{x}{\sqrt{x^7+77}} \geq \dfrac{1}{x^{6}}$ and the integral $\displaystyle\int_1^{\infty} \frac{1}{x^{6}}\,\mathrm{d}x = \infty$.

42. Set up the integral that will compute the area of the surface obtained by revolving the curve $x = (y-3)^2$ from $(0, 3)$ to $(1, 4)$ about the y-axis.