

## Section 8.2 Trigonometric Integrals.

- $\int \sin^m x \cos^n x \, dx$

1. if the power of cosine is odd, do the substitution  $u = \sin x$  (save one factor of  $\cos x$  and convert the rest to sine)
2. if the power of sine is odd, do the substitution  $u = \cos x$  (save one factor of  $\sin x$  and convert the rest to cosine)
3. if both  $m$  and  $n$  are even, use half-angle identities

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x) \quad \cos^2 x = \frac{1}{2}(1 + \cos 2x) \quad \sin x \cos x = \frac{1}{2} \sin 2x$$

- $\int \tan^m x \sec^n x \, dx$

1. if the power of secant is even, do the substitution  $u = \tan x$  (save one factor of  $\sec^2 x$  and convert the rest to tangent)
2. if the power of tangent is odd, do the substitution  $u = \sec x$  (save one factor of  $\sec x \tan x$  and convert the rest to secant)

- $\int \sin mx \cos nx \, dx, \int \sin mx \sin nx \, dx, \int \cos mx \cos nx \, dx$

use the corresponding identity:

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha - \beta) + \sin(\alpha + \beta)]$$

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

**Examples.** Find the integral

1.  $\int \sin^3 x \cos^4 x \, dx$

2.  $\int_0^{\pi/8} \sin^2(2x) \cos^3(2x) \, dx$

3.  $\int \sin^2 x \cos^4 x \, dx$

4.  $\int_0^{\pi/4} \tan^4 x \sec^2 x \, dx$

5.  $\int \tan x \sec^3 x \, dx$

6.  $\int \sin 3x \cos x \, dx$

**Review for Test 1.**

1. Let  $\mathcal{R}$  be the region in the first quadrant bounded by the curves  $y = x^3$  and  $y = 2x - x^2$ .
  - (a) Find the area of  $\mathcal{R}$
  - (b) Find the volume obtained by rotating  $\mathcal{R}$  about the  $y$ -axis
  - (c) Find the volume obtained by rotating  $\mathcal{R}$  about the  $x$ -axis
2. Find the volume of the solid obtained by rotating the region bounded by  $y = x$  and  $y = x^2$  about
  - (a) the  $x$ -axis
  - (b) the  $y$ -axis
  - (c) the line  $x = 4$
3. The base of solid  $S$  is the triangular region with vertices  $(0,0)$ ,  $(2,0)$ , and  $(0,1)$ . Cross-sections perpendicular to the  $x$ -axis are semicircles. Find the volume of  $S$ .
4. A heavy rope, 50 ft long, weighs 0.5 lb/ft and hangs over the edge of a building 120 ft high. How much work is done in pulling the rope to the top of the building?
5. A spring has a natural length of 20 cm. If a 10 J work is required to keep it stretched to a length 25 cm, how much work is done in stretching the spring from 30 cm to 80 cm?
6. A tank of water is 20 ft long and has a vertical cross section in a shape of an equilateral triangle with sides 2 ft long. The tank is filled with water to a depth of 18 inches. Determine the amount of work needed to pump all of the water to the top of the tank. The weight of water is 62.5 lb/ft<sup>3</sup>.
7. Find the average value of  $f = \sin^2 x \cos x$  on  $[-\pi/2, \pi/4]$ .
8. Evaluate the integral
  - (a)  $\int t^2 \cos(1 - t^3) dt$
  - (b)  $\int \frac{x^2}{\sqrt{1-x}} dx$
  - (c)  $\int_0^1 x^2 e^{-x} dx$
  - (d)  $\int x^3 e^{x^2} dx$
  - (e)  $\int \sqrt{t} \ln t dt$