Section 8.2 Trigonometric Integrals.

- $\int \sin^m x \, \cos^n x \, dx$
 - 1. if the power of cosine is odd, do the substitution $u = \sin x$ (save one factor of $\cos x$ and convert the rest to sine)
 - 2. if the power of sine is odd, do the substitution $u = \cos x$ (save one factor of $\sin x$ and convert the rest to cosine)
 - 3. if both m and n are even, use half-angle identities

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x) \qquad \cos^2 x = \frac{1}{2}(1 + \cos 2x) \qquad \sin x \cos x = \frac{1}{2}\sin 2x$$

- $\int \tan^m x \sec^n x \, dx$
 - 1. if the power of secant is even, do the substution $u = \tan x$ (save one factor of $\sec^2 x$ and convert the rest to tangent)
 - 2. if the power of tangent is odd, do the substitution $u = \sec x$ (save one factor of $\sec x \tan x$ and convert the rest to secant)
- $\int \sin mx \, \cos nx \, dx$, $\int \sin mx \, \sin nx \, dx$, $\int \cos mx \, \cos nx \, dx$ use the corresponding identity:

$$\sin \alpha \ \cos \beta = \frac{1}{2} \left[\sin(\alpha - \beta) + \sin(\alpha + \beta) \right]$$
$$\sin \alpha \ \sin \beta = \frac{1}{2} \left[\cos(\alpha - \beta) - \cos(\alpha + \beta) \right]$$
$$\cos \alpha \ \cos \beta = \frac{1}{2} \left[\cos(\alpha - \beta) + \cos(\alpha + \beta) \right]$$

Examples. Find the integral

1.
$$\int \sin^3 x \cos^4 x \, dx$$

2.
$$\int_{0}^{\pi/8} \sin^2(2x) \cos^3(2x) \, dx$$

4. $\int_{0}^{\pi/4} \tan^4 x \sec^2 x \ dx$

5. $\int \tan x \sec^3 x \, dx$



Review for Test 1.

- 1. Let \mathcal{R} be the region in the first quadrant bounded by the curves $y = x^3$ and $y = 2x x^2$.
 - (a) Find the area fo \mathcal{R}

(b) Find the volume obtained by rotating \mathcal{R} about the *y*-axis

(c) Find the volume obtained by rotating \mathcal{R} about the *x*-axis

- 2. Find the volume of the solid obtained by rotating the region bounded by y = x and $y = x^2$ about
 - (a) the x-axis

(b) the *y*-axis

(c) the line x = 4

3. The base of solid S is the triangular region with vertices (0,0), (2,0), and (0,1). Cross-sections perpendicular to the x-axis are semicircles. Find the volume of S.

4. A heavy rope, 50 ft long, weighs 0.5 lb/ft and hangs over the edge of a building 120 ft hight. How much work is done in pulling the rope to the top of the building?

5. A spring has a natural length of 20 cm. If a 10 J work is required to keep it stretched to a length 25 cm, how much work is done in stretching the spring from 30 cm to 80 cm?

6. A tank of water is 20 ft long and has a vertical cross section in a shape of an equilateral triangle with sides 2 ft long. The tank is filled with water to a depth of 18 inches. Determine the amount of work needed to pump all of the water to the top of the tank. The weight of water is 62.5 lb/ft^3 .

7. Find the average value of $f = \sin^2 x \cos x$ on $[-\pi/2, \pi/4]$.

8. Evaluate the integral

(a)
$$\int t^2 \cos(1-t^3) dt$$

(b)
$$\int \frac{x^2}{\sqrt{1-x}} dx$$



(d) $\int x^3 e^{x^2} dx$

(e) $\int \sqrt{t} \ln t \, dt$