## Section 8.2 Trigonometric Integrals.

- $\int \sin ^{m} x \cos ^{n} x d x$

1. if the power of cosine is odd, do the substitution $u=\sin x$ (save one factor of $\cos x$ and convert the rest to sine)
2. if the power of sine is odd, do the substitution $u=\cos x$ (save one factor of $\sin x$ and convert the rest to cosine)
3 . if both $m$ and $n$ are even, use half-angle identities

$$
\sin ^{2} x=\frac{1}{2}(1-\cos 2 x) \quad \cos ^{2} x=\frac{1}{2}(1+\cos 2 x) \quad \sin x \cos x=\frac{1}{2} \sin 2 x
$$

- $\int \tan ^{m} x \sec ^{n} x d x$

1. if the power of secant is even, do the substution $u=\tan x$ (save one factor of $\sec ^{2} x$ and convert the rest to tangent)
2. if the power of tangent is odd, do the substitution $u=\sec x$ (save one factor of $\sec x \tan x$ and convert the rest to secant)

- $\int \sin m x \cos n x d x, \int \sin m x \sin n x d x, \int \cos m x \cos n x d x$ use the corresponding identity:

$$
\begin{aligned}
\sin \alpha \cos \beta & =\frac{1}{2}[\sin (\alpha-\beta)+\sin (\alpha+\beta)] \\
\sin \alpha \sin \beta & =\frac{1}{2}[\cos (\alpha-\beta)-\cos (\alpha+\beta)] \\
\cos \alpha \cos \beta & =\frac{1}{2}[\cos (\alpha-\beta)+\cos (\alpha+\beta)]
\end{aligned}
$$

Examples. Find the integral

1. $\int \sin ^{3} x \cos ^{4} x d x$
2. $\int_{0}^{\pi / 8} \sin ^{2}(2 x) \cos ^{3}(2 x) d x$
3. $\int \sin ^{2} x \cos ^{4} x d x$
4. $\int_{0}^{\pi / 4} \tan ^{4} x \sec ^{2} x d x$
5. $\int \tan x \sec ^{3} x d x$
6. $\int \sin 3 x \cos x d x$

## Review for Test 1.

1. Let $\mathcal{R}$ be the region in the first quadrant bounded by the curves $y=x^{3}$ and $y=2 x-x^{2}$.
(a) Find the area fo $\mathcal{R}$
(b) Find the volume obtained by rotating $\mathcal{R}$ about the $y$-axis
(c) Find the volume obtained by rotating $\mathcal{R}$ about the $x$-axis
2. Find the volume of the solid obtained by rotating the region bounded by $y=x$ and $y=x^{2}$ about (a) the $x$-axis
(b) the $y$-axis
(c) the line $x=4$
3. The base of solid $S$ is the triangular region with vertices $(0,0),(2,0)$, and $(0,1)$. Cross-sections perpendicular to the $x$-axis are semicircles. Find the volume of $S$.
4. A heavy rope, 50 ft long, weighs $0.5 \mathrm{lb} / \mathrm{ft}$ and hangs over the edge of a building 120 ft hight. How much work is done in pulling the rope to the top of the building?
5. A spring has a natural length of 20 cm . If a 10 J work is required to keep it stretched to a length 25 cm , how much work is done in stretching the spring from 30 cm to 80 cm ?
6. A tank of water is 20 ft long and has a vertical cross section in a shape of an equilateral triangle with sides 2 ft long. The tank is filled with water to a depth of 18 inches. Determine the amount of work needed to pump all of the water to the top of the tank. The weight of water is $62.5 \mathrm{lb} / \mathrm{ft}^{3}$.
7. Find the average value of $f=\sin ^{2} x \cos x$ on $[-\pi / 2, \pi / 4]$.
8. Evaluate the integral
(a) $\int t^{2} \cos \left(1-t^{3}\right) d t$
(b) $\int \frac{x^{2}}{\sqrt{1-x}} d x$
(c) $\int_{0}^{1} x^{2} e^{-x} d x$
(d) $\int x^{3} e^{x^{2}} d x$
(e) $\int \sqrt{t} \ln t d t$
