Math 152/172 **WEEK in REVIEW 5** **Spring 2017**

Sections 8.3, 8.4, 8.9

Section 8.3: Trigonometric Substitutions

Expression	Substitution	Identity
$\sqrt{a^2 - x^2}$	$x = a\sin t, \quad -\frac{\pi}{2} \le t \le \frac{\pi}{2}$	$1 - \sin^2 t = \cos^2 t$
$\sqrt{a^2 + x^2}$	$x = a\tan t, \quad -\frac{\pi}{2} < t < \frac{\pi}{2}$	$1 + \tan^2 t = \sec^2 t$
$\sqrt{x^2 - a^2}$	$x = a\sec t, \quad 0 \le t \le \frac{\pi}{2}$ or $\pi \le t \le \frac{3\pi}{2}$	$\sec^2 t - 1 = \tan^2 t$

$\int \sec t\,dt = \ln|\sec t + \tan t| + C$

$\int \sec^3 t\,dt = \frac{1}{2}(\sec t \tan t + \ln|\sec t + \tan t|) + C$

Problems.

1. Evaluate the integral

(a) $\displaystyle\int \frac{x^2}{\sqrt{4 - x^2}}\,dx$

$4 = 2^2$

$x = 2\tan t \Rightarrow \tan t = \frac{x}{2}$

$dx = 2\sec^2 t\,dt$

$\sqrt{4+x^2} = \sqrt{4 + 4\tan^2 t}$

$= \sqrt{4(1+\tan^2 t)}$

$= \sqrt{4\sec^2 t}$

$\sqrt{4+x^2} = 2\sec t$

$\sec t = \frac{\sqrt{4+x^2}}{2}$

$= \int \frac{4\tan^2 t\, \cancel{2}\sec^{\cancel{2}} t\,dt}{\cancel{2\sec t}} = 4\int \overbrace{\tan^2 t}^{\sec^2 t - 1} \sec t\,dt$

$= 4\int (\sec^2 t - 1)\sec t\,dt = 4\int \sec^3 t\,dt - 4\int \sec t\,dt$

$= \frac{4}{2}(\sec t \tan t + \ln|\sec t + \tan t|) - 4\ln|\sec t + \tan t| + C$

$= 2\sec t \tan t - \ln|\sec t + \tan t| + C$

$= \boxed{2 \cdot \frac{\sqrt{4+x^2}}{2}\,\frac{x}{2} - \ln\left|\frac{\sqrt{4+x^2}}{2} + \frac{x}{2}\right| + C}$

$u = 6x - x^2$
$du = 6 - 2x$

(b) $\displaystyle\int \frac{x}{\sqrt{6x - x^2}}\, dx$

complete the square:

$6x - x^2 = -(x^2 - 6x) = -(x^2 - 2\cdot 3\cdot x + 3^2 - 3^2) = -(\overbrace{x^2 - 6x + 9}^{(x-3)^2} - 9) = -((x-3)^2 - 9) = 9 - (x-3)^2$

$$= \int \frac{x}{\sqrt{9 - (x-3)^2}}\, dx$$

$x - 3 = 3\sin t \Rightarrow t = \sin^{-1}\left(\frac{x-3}{3}\right)$

$x = 3\sin t + 3$

$dx = 3\cos t\, dt$

$\sqrt{9 - 9\sin^2 t} = \sqrt{9(1 - \sin^2 t)}$

$= \sqrt{9\cos^2 t}$

$\sqrt{9 - (x-3)^2} = 3\cos t$

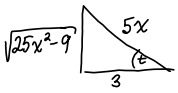
$3\cos t = \sqrt{9 - (x-3)^2}$

$$= \int \frac{3\sin t + 3}{\cancel{3\cos t}}\, \cancel{3\cos t}\, dt$$

$$= \int (3\sin t + 3)\, dt = -3\cos t + 3t + C$$

$$= \boxed{-\sqrt{9 - (x-3)^2} + 3\cdot \sin^{-1}\left(\frac{x-3}{3}\right) + C}$$

(c) $\displaystyle\int \frac{dx}{x^2\sqrt{25x^2-9}} = \int \frac{dx}{x^2\sqrt{(5x)^2-3^2}}$



$\sec t = \frac{5x}{3}$

$\cos t = \frac{3}{5x}$

$\sin t = \frac{\sqrt{25x^2-9}}{5x}$

$5x = 3\sec t$

$x = \frac{3}{5}\sec t$

$dx = \frac{3}{5}\sec t \tan t \, dt$

$\sqrt{25x^2-9} = \sqrt{9\sec^2 t - 9}$

$= \sqrt{9(\sec^2 t - 1)}$

$= \sqrt{9\tan^2 t}$

$\sqrt{25x^2-9} = 3\tan t$

$$= \int \frac{\cancel{\frac{3}{5}}\cancel{\sec t}\,\cancel{\tan t}\,dt}{\frac{9}{25}\sec^{\cancel{2}} t\,\cancel{3\tan t}}$$

$$= \frac{1}{5}\cdot\frac{25}{9}\int \frac{dt}{\sec t} = \frac{5}{9}\int \cos t\,dt$$

$$= \frac{5}{9}\sin t + C$$

$$= \boxed{\frac{\cancel{5}}{9}\,\frac{\sqrt{25x^2-9}}{\cancel{5}x} + C}$$

Section 8.4: Integration Of Rational Functions By Partial Fractions

- Rational function $f(x) = \dfrac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are polynomials.
- Make sure that the fraction is **proper**, that is the degree of the numerator is strictly less than the degree of denominator. Otherwise, do long divisions.
- Factor out the denominator $Q(x)$ as far as you can.
- For each factor in the denominator use the following table:

Factor	**Corresponding Partial Fraction**
linear factor $ax+b$	$\dfrac{A}{ax+b}$
linear repeated factor $(ax+b)^r$	$\dfrac{A_1}{ax+b} + \dfrac{A_2}{(ax+b)^2} + \cdots + \dfrac{A_r}{(ax+b)^r}$
quadratic irreducible factor ax^2+bx+c, $b^2-4ac<0$	$\dfrac{Ax+B}{ax^2+bx+c}$
quadratic irreducible factor $(ax^2+bx+c)^r$, $b^2-4ac<0$	$\dfrac{A_1x+B_1}{ax^2+bx+c} + \dfrac{A_2x+B_2}{(ax^2+bx+c)^2} + \cdots + \dfrac{A_rx+B_r}{(ax^2+bx+c)^r}$

- Solve for numerators using roots of the denominator and/or matching powers of x. Then integrate.

Problems.

3. Decompose into partial fractions the rational function without computing coefficient of the decomposition:

(a) $\dfrac{7}{x(x^3-2x^2+2x)} = \dfrac{7}{x^2\underbrace{(x^2-2x+2)}_{\text{quadratic irreducible}}} = \boxed{\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{Cx+D}{x^2-2x+2}}$

(b) $\dfrac{x-1}{(x+2)^3(x^2-2x+5)^2} = \boxed{\dfrac{A}{x+2} + \dfrac{B}{(x+2)^2} + \dfrac{C}{(x+2)^3} + \dfrac{Dx+E}{x^2-2x+5} + \dfrac{Fx+G}{(x^2-2x+5)^2}}$

$(-2)^2-4(5)=-16<0$

4. Evaluate the integral

(a) $\displaystyle\int \frac{7}{(x-2)(x+5)}\,dx$

$$\frac{7}{(x-2)(x+5)} = \frac{A}{x-2} + \frac{B}{x+5}$$

$$\frac{7}{\cancel{(x-2)(x+5)}} = \frac{A(x+5) + B(x-2)}{\cancel{(x-2)(x+5)}}$$

$$7 = A(x+5) + B(x-2)$$

$x=-5$: $7 = 0 + B(-5-2)$

$7 = -7B \Rightarrow \boxed{B=-1}$

$x=2$: $7 = A(2+5)+0$

$7 = 7A \Rightarrow A = 1$

$$\int \frac{7\,dx}{(x-2)(x+5)} = \int\left(\frac{1}{x-2} - \frac{1}{x+5}\right)dx$$

$$= \ln|x-2| - \ln|x+5| + C$$

$$= \boxed{\ln\left|\frac{x-2}{x+5}\right| + C}$$

(b) $\displaystyle\int \frac{x^5}{(x-2)^2}\,dx$

$\dfrac{x^5}{x^2-4x+4}$ improper fraction

$$
\begin{array}{r|l}
 & x^3+4x^2+12x+32 \\
\hline
x^2-4x+4 & x^5 \\
 & -\,x^5-4x^4+4x^3 \\
 & \quad 4x^4-4x^3 \\
 & \quad -\,4x^4-16x^3+16x^2 \\
 & \qquad 12x^3-16x^2 \\
 & \qquad -\,12x^3-48x^2+48x \\
 & \qquad\quad 32x^2-48x \\
 & \qquad\quad -\,32x^2-128x+128 \\
 & \qquad\qquad 80x-128
\end{array}
$$

$$\frac{x^5}{x^2-4x+4} = x^3+4x^2+12x+32+\underbrace{\frac{80x-128}{x^2-4x+4}}$$

$$\frac{80x-128}{(x-2)^2} = 16\,\frac{5x-8}{(x-2)^2} = \frac{A}{x-2}+\frac{B}{(x-2)^2}$$

$$16\,\frac{5x-8}{\cancel{(x-2)^2}} = \frac{A(x-2)+B}{\cancel{(x-2)^2}}$$

$$80x-128 = A(x-2)+B$$

$x=2$: $160-128=B \Rightarrow \boxed{B=32}$

$x=0$: $-128=-2A+B$

$2A = B+128$

$A = \dfrac{B+128}{2} = \dfrac{32+128}{2} = \boxed{80=A}$

$$\int \frac{x^5\,dx}{x^2-4x+4} = \int\left[x^3+4x^2+12x+32+\frac{80}{x-2}+\frac{32}{(x-2)^2}\right]dx$$

$$= \boxed{\frac{x^4}{4}+\frac{4x^3}{3}+\frac{12x^2}{2}+32x+80\ln|x-2|-\frac{32}{x-2}+C}$$

(c) $\displaystyle\int \frac{x^2-3x+7}{\underbrace{(x-1)}_{\text{linear}}\underbrace{(x^2+1)}_{\text{quadratic irreduable}}}\,dx$

$$\frac{x^2-3x+7}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

$$\frac{x^2-3x+7}{\cancel{(x-1)(x^2+1)}} = \frac{A(x^2+1)+(Bx+C)(x-1)}{\cancel{(x-1)(x^2+1)}}$$

$$x^2-3x+7 = Ax^2+A+Bx^2-Bx+Cx-C$$

$$x^2-3x+7 = x^2(A+B)+x(-B+C)+(A-C)$$

$$\begin{array}{ll} x^2: & \\ x: & \\ 1: & \end{array}\begin{cases} 1 = A+B \Rightarrow B = 1-A \\ -3 = -B+C \\ 7 = A-C \Rightarrow C = A-7 \end{cases}$$

$$-3 = -\overbrace{(1-A)}^{B} + \overbrace{A-7}^{C}$$

$$-3 = -1+A+A-7$$

$$2A = 5 \Rightarrow \boxed{A = \frac{5}{2}}$$

$$B = 1-A = 1-\frac{5}{2} = \boxed{-\frac{3}{2} = B}$$

$$C = A-7 = \frac{5}{2}-7 = \frac{5-14}{2} = \boxed{-\frac{9}{2} = C}$$

$$\int \frac{x^2-3x+7}{(x-1)(x^2+1)}\,dx = \int\left[\frac{5}{2}\,\frac{1}{x-1} + \frac{-\frac{3}{2}x-\frac{9}{2}}{x^2+1}\right]dx = \frac{5}{2}\int\frac{dx}{x-1} + \int\frac{-3/2\,x}{x^2+1}\,dx + \int\frac{-9/2}{x^2+1}\,dx$$

$u = x^2+1$

$du = 2x\,dx \Rightarrow x\,dx = \frac{du}{2}$

$$= \frac{5}{2}\ln|x-1| - \frac{3}{2}\int\frac{\frac{du}{2}}{u} - \frac{9}{2}\int\frac{dx}{x^2+1}$$

$$= \frac{5}{2}\ln|x-1| - \frac{3}{4}\ln|u| - \frac{9}{2}\arctan x + C$$

$$= \boxed{\frac{5}{2}\ln|x-1| - \frac{3}{4}\ln|x^2+1| - \frac{9}{2}\arctan x + C}$$

(d) $\displaystyle\int \frac{dx}{(x^2+1)(x^2+x+1)}$

$$\frac{1}{(x^2+1)(x^2+x+1)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+x+1}$$

$$\frac{1}{\cancel{(x^2+1)(x^2+x+1)}} = \frac{(Ax+B)(x^2+x+1)+(Cx+D)(x^2+1)}{\cancel{(x^2+1)(x^2+x+1)}}$$

$$0\cdot x^3+0\cdot x^2+0\cdot x+1 = Ax^3+Ax^2+Ax+Bx^2+Bx+B+Cx^3+Cx+Dx^2+D$$
$$= x^3(A+C)+x^2(A+B+D)+x(A+B+C)+(B+D)$$

x^3: $0 = A+C$ $\quad \boxed{C=1}$

x^2: $0 = A+B+D \Rightarrow \boxed{A=-1}$

x: $0 = A+B+C \Rightarrow \boxed{B=0}$

1: $1 = B+D \Rightarrow \boxed{D=1}$

$0 = A+B+C$

$0 = \underbrace{(A+C)}_{0}+B \Rightarrow 0 = 0+B$

$$\int \frac{1\,dx}{(x^2+1)(x^2+x+1)} = \int\left[\frac{-x}{x^2+1} + \frac{x+1}{x^2+x+1}\right]dx$$

$$-\int \frac{x}{x^2+1}\,dx \quad \left|\begin{array}{l} u = x^2+1 \\ du = 2x\,dx \\ x\,dx = \frac{du}{2}\end{array}\right| = -\int \frac{\frac{du}{2}}{u} = -\frac{1}{2}\int\frac{du}{u} = -\frac{1}{2}\ln|u|+C = -\frac{1}{2}\ln|x^2+1|+C$$

$$\int \frac{x+1}{x^2+x+1}\,dx = \int \frac{x+1}{(x+1/2)^2+3/4}\,dx \quad \left|\begin{array}{l} u = x+1/2 \Rightarrow x = u-1/2 \\ du = dx\end{array}\right| = \int \frac{u-1/2+1}{u^2+\frac{3}{4}}\,du = \int \frac{u+1/2}{u^2+\frac{3}{4}}\,du$$

Complete the square:

$$x^2+x+1 = x^2+2\cdot x\cdot\frac{1}{2}+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1$$

$$x^2+x+1 = \left(x+\frac{1}{2}\right)^2+\frac{3}{4}$$

$$\boxed{\int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\frac{x}{a}+C}$$

$$= \int \frac{u}{u^2+\frac{3}{4}}\,du + \frac{1}{2}\int\frac{du}{u^2+3/4} \quad \left|\begin{array}{l} v = u^2+3/4 \\ dv = 2u\,du\end{array}\right|$$

$$= \frac{1}{2}\int\frac{dv}{v} + \frac{1}{2}\int\frac{du}{u^2+3/4}$$

$$= \frac{1}{2}\ln|v| + \frac{1}{2}\,\frac{1}{\frac{\sqrt{3}}{2}}\arctan\frac{u}{\frac{\sqrt{3}}{2}} + C$$

$$= \frac{1}{2}\ln\left|u^2+\frac{3}{4}\right| + \frac{1}{\sqrt{3}}\arctan\frac{2u}{\sqrt{3}} + C$$

$$= \frac{1}{2}\ln\left|\left(x+\frac{1}{2}\right)^2+\frac{3}{4}\right| + \frac{1}{\sqrt{3}}\arctan\frac{2\left(x+\frac{1}{2}\right)}{\sqrt{3}} + C$$

$$\int \frac{dx}{(x^2+1)(x^2+x+1)} = \boxed{-\frac{1}{2}\ln|x^2+1| + \frac{1}{2}\ln\left|\left(x+\frac{1}{2}\right)^2+\frac{3}{4}\right| + \frac{1}{\sqrt{3}}\arctan\frac{2x+1}{\sqrt{3}} + C}$$

Section 8.9: Improper Integrals

- TYPE I: Infinite Interval and Continuous Integrand.
  - $\int_a^{\infty} f(x)dx = \lim_{t\to\infty} \int_a^t f(x)dx$
  - $\int_{-\infty}^{b} f(x)dx = \lim_{t\to-\infty} \int_t^b f(x)dx$
  - $\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{a} f(x)dx + \int_a^{\infty} f(x)dx$ where a is any real number
- TYPE II: Discontinuous Integrand and Finite Interval:
  - if f is discontinuous at b, then $\int_a^b f(x)dx = \lim_{t\to b^-} \int_a^t f(x)dx$
  - if f is discontinuous at a, then $\int_a^b f(x)dx = \lim_{t\to a^+} \int_t^b f(x)dx$
  - if f has discontinuity at c $(a < c < b)$, then $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$
- $\boxed{\int_1^{\infty} \frac{dx}{x^p} = \begin{cases} \frac{1}{p-1}, & \text{if } p > 1 \\ \text{diverges}, & \text{if } p \le 1 \end{cases}}$ $\boxed{\int_0^1 \frac{dx}{x^p} = \begin{cases} \frac{1}{1-p}, & \text{if } p < 1 \\ \text{diverges}, & \text{if } p \ge 1 \end{cases}}$
- **Comparison theorem.** Suppose that f and g are continuous functions with $f(x) \ge g(x) \ge 0$ for $x \ge a$.
  1. If $\int_a^{\infty} f(x)dx$ is convergent, then $\int_a^{\infty} g(x)dx$ is convergent.
  2. If $\int_a^{\infty} g(x)dx$ is divergent, then $\int_a^{\infty} f(x)dx$ is divergent.

5. Compute the following integrals or show that they diverge.

(a) $\displaystyle\int_e^\infty \frac{dx}{x\ \ln^5 x} = \lim_{t\to\infty} \int_e^t \frac{dx}{x\ln^5 x}$

$u = \ln x$

$du = \dfrac{dx}{x}$

$x = e \Rightarrow u = \ln e = 1$

$x = t \Rightarrow u = \ln t$

$\displaystyle = \lim_{t\to\infty} \int_1^{\ln t} \frac{du}{u^5} = \lim_{t\to\infty} \frac{u^{-5+1}}{-5+1}\Bigg|_1^{\ln t} = -\frac{1}{4}\left[\cancel{\lim_{t\to\infty} (\ln t)^{-4}}^{\,0} - 1\right]$

$= \boxed{\frac{1}{4}}$ convergent

$\lim_{t\to\infty} \ln t = \infty$

(b) $\displaystyle\int_{-\infty}^{0} (1+x)e^x\,dx = \lim_{t\to-\infty} \int_t^0 (1+x)e^x\,dx = \left| \begin{array}{ll} u = 1+x & v' = e^x \\ u' = 1 & v = e^x \end{array} \right|$

$\lim_{x\to\infty} e^x = \infty$

$\lim_{x\to-\infty} e^x = 0$

$= \lim_{t\to-\infty} \left[(1+x)e^x \Big|_t^0 - \int_t^0 e^x\,dx \right] = \lim_{t\to-\infty} \left(e^0 - (1+t)e^t - e^x\Big|_t^0 \right) = \lim_{t\to-\infty} \left(\cancel{1} - (1+t)e^t - \cancel{e^0} + \cancel{e^t}^{\,0} \right)$

$= -\lim_{t\to-\infty} (1+t)e^t = |0\cdot\infty| = -\lim_{t\to-\infty} \frac{1+t}{e^{-t}} = -\lim_{t\to-\infty} \frac{1}{-e^{-t}} = \lim_{t\to-\infty} e^t = \boxed{0}$

convergent.

(c) $\displaystyle\int_{-\infty}^{\infty} \frac{5x^4}{(x^5+3)^3}dx = \int_{-\infty}^{0} + \int_0^{\infty} = \lim_{t\to-\infty} \int_t^0 \frac{5x^4}{(x^5+3)^3}\,dx + \lim_{s\to\infty} \int_0^s \frac{5x^4}{(x^5+3)^3}\,dx$

must converge.

$\frac{x^4}{(x^5)^3} = \frac{x^4}{x^{15}} = \frac{1}{x^{11}}$ $\quad p = 11 > 1$

$= -\frac{1}{2} \lim_{t\to-\infty} \frac{1}{(x^5+3)^2}\Big|_t^0 - \frac{1}{2} \lim_{s\to\infty} \frac{1}{(x^5+3)^2}\Big|_0^s$

$= -\frac{1}{2}\cdot\frac{1}{3^2} + \frac{1}{2}\cancel{\lim_{t\to-\infty} \frac{1}{(t^5+3)^2}}^{\,0} - \frac{1}{2}\cancel{\lim_{s\to\infty} \frac{1}{(s^5+3)^2}}^{\,0} + \cancel{\frac{1}{2\cdot 3^2}}$

$= \boxed{0}$ convergent

$\int \frac{5x^4}{(x^5+3)^3}\,dx \quad \left| \begin{array}{l} u = x^5+3 \\ du = 5x^4\,dx \end{array} \right.$

$= \int \frac{du}{u^3} = \frac{u^{-3+1}}{-3+1} + C$

$= -\frac{1}{2u^2} + C$

$= -\frac{1}{2(x^5+3)^2} + C$

(d) $\displaystyle\int_0^9 \frac{dx}{\sqrt[3]{x-4}}$ discontinuity @ $x=4$

$\frac{1}{x^{1/3}}$ $\quad p = 1/3 < 1 \Rightarrow$ must converge.

$\int_0^9 \frac{dx}{\sqrt[3]{x-4}} = \int_0^4 + \int_4^9 = \lim_{t\to4^-} \int_0^t \frac{dx}{(x-4)^{1/3}} + \lim_{s\to4^+} \int_s^9 \frac{dx}{(x-4)^{1/3}}$

$= \lim_{t\to4^-} \frac{3}{2}(x-4)^{2/3}\Big|_0^t + \lim_{s\to4^+} \frac{3}{2}(x-4)^{2/3}\Big|_s^9$

$= \frac{3}{2}\cancel{\lim_{t\to4^-} (t-4)^{2/3}}^{\,0} - \frac{3}{2}(-4)^{2/3} + \frac{3}{2}(9-4)^{2/3} - \frac{3}{2}\cancel{\lim_{s\to4^+} (s-4)^{2/3}}^{\,0} = \boxed{\frac{3}{2}\left(\sqrt[3]{25} - \sqrt[3]{16}\right)}$

$\int \frac{dx}{(x-4)^{1/3}} = \frac{(x-4)^{-1/3+1}}{-1/3+1} + C$

$= \frac{(x-4)^{2/3}}{2/3} + C = \frac{3}{2}(x-4)^{2/3} + C$

6. Determine whether the given integrals converge or diverge using the Comparison Theorem.

$\lim_{x\to\infty} e^{-x} = 0$

$\lim_{x\to-\infty} e^{-x} = \infty$

(a) $\displaystyle\int_0^\infty \frac{dx}{x^7+e^{7x}}$ – $\boxed{\text{converges}}$ by the Comparison Thm (part 1).

$$\frac{1}{x^7+e^{7x}} \le \frac{1}{e^{7x}}$$

$$\int_0^\infty e^{-7x}\,dx = \lim_{t\to\infty}\int_0^t e^{-7x}\,dx = \lim_{t\to\infty}\left(-\frac{1}{7}e^{-7x}\Big|_0^t\right) = -\frac{1}{7}\lim_{t\to\infty} e^{-7t} + \frac{1}{7}e^0 = \frac{1}{7}$$

$e^{-7t} \to 0$

convergent.

(b) $\displaystyle\int_5^\infty \frac{x^2}{x^{5/2}-x}\,dx$ – $\boxed{\text{diverges}}$ by the Comparison Thm (part 2)

$$\frac{x^2}{x^{5/2}} = \frac{1}{x^{5/2-2}} = \frac{1}{x^{1/2}} \qquad p=\frac{1}{2}<1$$

$$\frac{1}{x^{1/2}} = \frac{x^2}{x^{5/2}} \le \frac{x^2}{x^{5/2}-x}$$

$\displaystyle\int_5^\infty \frac{dx}{x^{1/2}}$ is divergent

(c) $\displaystyle\int_{10}^\infty \frac{\sin^4(7x)}{x^7}\,dx$ $\boxed{\text{converges}}$ by the Comparison Thm (part 1)

$$\underbrace{0 \le \frac{\sin^4(7x)}{x^7} \le \frac{1}{x^7}}$$

$\displaystyle\int_{10}^\infty \frac{dx}{x^7}$ is convergent, since $p=7>1$