Section 9.3. Arc Length

- If a curve C is defined by the equations $x = x(t), y = y(t), \quad a \le t \le b$, then $L = \int\limits_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2}dt$
- If a curve C is given by the equation $y = y(x), \quad a \le x \le b$, then $L = \int\limits_a^b \sqrt{1 + [y'(x)]^2}dx$
- If a curve C is given by the equation $x = x(y), \quad c \le y \le d$, then $L = \int\limits_c^d \sqrt{1 + [x'(y)]^2}dy$

Problems:

1. Find the length of the curve $x = \cos^3 t, y = \sin^3 t$, $0 \le t \le \pi/2$.

2. Find the length of the curve $x = \dfrac{1}{4}\ln y - \dfrac{1}{2}y^2$ from $y = 1$ to $y = e$.

3. A wire hanging between two poles (at $x = -10$ and $x = 10$) takes the shape of a catenary with equation

$$y = 2(e^{x/4} + e^{-x/4}).$$

Find the length of the wire.

Section 9.4. Area of a Surface of Revolution

For rotation about the x-axis, the surface area formulas are:

- if a curve is given as $y = y(x)$, $a \le x \le b$, then $S.A. = 2\pi \int\limits_a^b y(x)\sqrt{1 + [y'(x)]^2}dx$
- if a curve is described as $x = x(y)$, $c \le y \le d$, then $S.A. = 2\pi \int_c^d y\sqrt{1 + [x'(y)]^2}dy$
- if a curve is defined by $x = x(t)$,$y = y(t)$, $a \le t \le b$, then $S.A. = \int\limits_a^b y(t)\sqrt{[x'(t)]^2 + [y'(t)]^2}dt$

For rotation about the y-axis, the surface area formulas are:

- if a curve is given as $y = y(x)$, $a \le x \le b$, then $S.A. = 2\pi \int\limits_a^b x\sqrt{1 + [y'(x)]^2}dx$
- if a curve is described as $x = x(y)$, $c \le y \le d$, then $S.A. = 2\pi \int_c^d x(y)\sqrt{1 + [x'(y)]^2}dy$
- if a curve is defined by $x = x(t)$,$y = y(t)$, $a \le t \le b$, then $S.A. = \int\limits_a^b x(t)\sqrt{[x'(t)]^2 + [y'(t)]^2}dt$

Problems:

4. The curve $y = x^2$, $0 \le x \le 1$, is rotated about the y-axis. Find the area of the resulting surface.

5. The curve $x = 1 - \cos(2t)$, $y = 2t + \sin(2t)$, $0 \le t \le \pi/4$ is rotated about the x-axis. Find the area of the resulting surface.

6. Set up (but don't evaluate) the integral that gives the surface area obtaine by rotating the curve

$$x = \sin(\pi y^2/8), \quad 1 \le y \le 2,$$

(a) about the x-axis

(b) about the y-axis

7. The curve $x = \sin(at), y = \cos(at), 0 \le t \le \frac{\pi}{2a}$ is rotated about the x-axis (here a is an arbitrary positive constant). Find the area of the resulting surface.

Section 10.1. Sequences

- If $\lim_{n\to\infty} a_n$ exists and finite then we say that the sequence $\{a_n\}$ **converges**. Otherwise, we say the sequence **diverges**. (Recall all techniques for finding limits at infinity.)
- **The Squeeze Theorem for Sequences:** If $a_n \le b_n \le c_n$ for all n and $\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = L$, then $\lim_{n\to\infty} b_n = L$.
- If $\lim_{n\to\infty} |a_n| = 0$, then $\lim_{n\to\infty} a_n = 0$.
- $\{a_n\}$ **increasing**: show that $a_{n+1} - a_n > 0$, or $f'(x) > 0$ (where $f(n) = a - n$); or $\dfrac{a_{n+1}}{a_n} > 1$ (provided $a_n > 0$ for all n.) Note: reverse signs for $\{a_n\}$ **decreasing**.

Problems:

8. Define the n-th term of the sequence $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \dots\right\}$ and find its limit.

9. Determine if the given sequence converges or diverges. If it converges, find the limit.

(a) $a_n = \dfrac{3n^5 - 12n^3 + 2012}{2012 - 12n^4 - 4n^4 - 9n^5}$

(b) $b_n = \dfrac{3n^5 - 12n^3 + 2012}{2012 - 12n^4 - 4n^4 - 9n^5 + 11n^6}$

(c) $c_n = \dfrac{12n^7 + 2012}{2012 - 12n^4 - 4n^5 - 9n^6}$

10. Determine if the sequence with the given general term ($n \geq 1$) converges or diverges. If it converges, find the limit.

(a) $a_n = \ln(n^2 + 3) - \ln(7n^2 - 5)$

(b) $z_n = \dfrac{1}{n^4} \sin\left(\dfrac{1}{n^5}\right)$

(c) $y_n = \dfrac{(-1)^n}{n^3}$

(d) $x_n = \dfrac{(-1)^n n}{3n + 33}$

11. Assuming that the sequence defined recursively by $a_n = 1$, $a_{n+1} = \dfrac{1}{2}\left(a_n + \dfrac{9}{a_n}\right)$ is convergent, find its limit.

12. Determine whether the given sequence is increasing or decreasing.

   (a) $\{\arctan(n)\}_{n=1}^{\infty}$

   (b) $\{n - 2^n\}_{n=1}^{\infty}$