Review for Test 2 over 8.3, 8.4, 8.9, 9.3, 9.4, 10.1

1. Write out the form of the partial fraction decomposition (do not try to solve)

$$\frac{20x^3 + 12x^2 + x}{(x^3 - x)(x^3 + 2x^2 - 3x)(x^2 + x + 1)(x^2 + 9)^2}$$

2. Evaluate the integral

   (a) $\displaystyle\int (4x^2 - 25)^{-3/2}\, dx$

(b) $\displaystyle\int \frac{(x-1)^2}{5\sqrt{24-x^2-2x}}dx$

(c) $\displaystyle\int \frac{5x^2 + x + 12}{x^3 + 4x} dx$

3. Determine whether the given integral is convergent or divergent.

(a) $\displaystyle\int_1^\infty \frac{4+\cos^4 x}{x}\,dx$

(b) $\displaystyle\int_1^\infty \frac{3+\sin x}{x^2}\,dx$

(c) $\displaystyle\int_0^\infty \frac{1}{\sqrt{x}+e^{4x}}\,dx$

4. Compute the following integrals or show that they diverge.

(a) $\displaystyle\int_{e}^{\infty} \frac{dx}{x\ \ln^5 x}$

(b) $\displaystyle\int_{-\infty}^{0} (1+x)e^{x}\ dx$

(c) $\displaystyle\int_{-\infty}^{\infty} \frac{5x^4}{(x^5+3)^3}\,dx$

(d) $\displaystyle\int_{0}^{2017} \frac{1}{\sqrt{2017-x}}\,dx$

5. The curve $y = \sin x$ for $0 \le x \le \pi$ is rotated about the x-axis. Set up, *but don't evaluate* the integral for the area of the resulting surface.

6. The curve $y = \dfrac{1}{2}\left(e^x + e^{-x}\right)$, $0 \le x \le 1$, is rotated about the x-axis. Find the area of the resulting surface.

7. Set up, *but don't evaluate* the integral for the length of the curve $x = 2t^2$, $y = t^3$, $0 \leq t \leq 1$.

8. Find length of the curve $y = \dfrac{1}{\pi} \ln(\sec(\pi x))$ from the point $(0, 0)$ to the point $\left(\dfrac{1}{6}, \ln \dfrac{2}{\sqrt{3}}\right)$.

9. Determine if the sequence $\{a_n\}_{n=2}^{\infty}$ is decreasing and bounded:

(a) $a_n = \ln n$

(b) $a_n = \cos n^2$

(c) $a_n = e^{-n}$

(d) $a_n = e^n + 11$

(e) $a_n = 1 - \frac{1}{n^2}$

10. Determine if the sequence converges or diverges. If converges, find its limit.

(a) $\left\{ \dfrac{2017 + (-1)^n}{n^{2017}} \right\}_{n=1}^{\infty}$

(b) $\left\{ \dfrac{7n + 6n^3 + n^2}{(n+3)(n^2+8)} \right\}_{n=4}^{\infty}$

11. Assuming that the sequence defined recursively by $a_1 = 1$, $a_{n+1} = \dfrac{1}{2}\left(a_n + \dfrac{16}{a_n}\right)$ is convergent, find its limit.