Math171-200 Sample Problems for the Final Spring 2012

1. Given vectors $\vec{a} = \vec{\imath} - 2\vec{\jmath}$, $\vec{b} =< -2, 3 >$. Find
   (a) a unit vector $\vec{u}$ that has the same direction as $2\vec{b} + \vec{a}$.
   (b) angle between $\vec{a}$ and $\vec{b}$
   (c) $\text{comp}_{\vec{b}}\vec{a}$, $\text{proj}_{\vec{b}}\vec{a}$.
2. Find the work done by by a force of 20 lb acting in the direction N50$^{\text{O}}$W in moving an object 4 ft due west.
3. Find the distance from the point (-2,3) to the line $3x - 4y + 5 = 0$.
4. Find vector and parametric equations for the line passing through the points $A(1, -3)$ and $B(2, 1)$.
5. Find all points of discontinuity for the function

$$f(x) = \begin{cases} x^2 + 1 & , \text{ if } x < 2, \\ x + 2 & , \text{ if } x \geq 2. \end{cases}$$

6. Find the vertical and horizontal asymptotes of the curve $y = \dfrac{x^2 + 4}{3x^2 - 3}$.
7. Find $\dfrac{dy}{dx}$ for each function
   (a) $y = (\sin x)^x$.
   (b) $y = \dfrac{\sqrt[5]{2x - 1}(x^2 - 4)^2}{\sqrt[3]{1 + 3x}}$
   (c) $y(t) = \sin^{-1} t$, $x(t) = \cos^{-1}(t^2)$.
   (d) $2x^2 + 2xy + y^2 = x$.
8. Find the equation of the tangent line to the curve $y = x\sqrt{5 - x}$ at the point (1,2).
9. A particle moves on a vertical line so that its coordinate at time t is $y = t^3 - 12t + 3$, $t \geq 0$.
   (a) Find the velocity and acceleration functions.
   (b) When is the particle moving upward?
   (c) Find the distance that particle travels in the time interval $0 \leq t \leq 3$
10. The vector function $\vec{r}(t) =< t, 25t - 5t^2 >$ represents the position of a particle at time t. Find the velocity, speed, and acceleration at $t = 1$.
11. Find y'' if $y = \mathrm{e}^{-5x} \cos 3x$
12. Find $\dfrac{d^{50}}{dx^{50}} \cos 2x$

13. Use differentials to estimate $(1.09)^{10}$.

14. The volume of a cube is increasing at a rate of $10\text{cm}^3/\text{min}$. How fast is the surface area increasing when the edge length is 30 cm?

15. If $f(x) = x + x^2 + e^x$ and $g(x) = f^{-1}(x)$, find $g'(1)$.

16. Solve the equation $\ln(x+6) + \ln(x-3) = \ln 5 + \ln 2$

17. Find $\cos^{-1}\left(\sin \dfrac{5\pi}{4}\right)$.

18. Evaluate each limit:

    (a) $\displaystyle\lim_{x\to 0} \frac{\sin x + \sin 2x}{\sin 3x}$

    (b) $\displaystyle\lim_{x\to 0} (\cot x - \csc x)$

    (c) $\displaystyle\lim_{x\to 0} x^{\sin x}$

19. Find the absolute maximum and absolute minimum values of $f(x) = x^3 - 2x^2 + x$ on [-1,1].

20. For the function $y = x^4 - 6x^2$ find

    (a) Intervals on which the function is increasing/decreasing.

    (b) All local minima/local maxima.

    (c) Intervals on which the function is CU/CD.

    (d) Inflection points.

21. The top and the bottom margins of a poster are each 6 cm and the side margins are each 4 cm. If the area of the printed material on the poster is fixed at 384 cm^2, find the dimensions of the poster with the smallest total area.

22. Evaluate the sum $\sum\limits_{i=0}^{n} (2^i + i^2)$

23. Find the derivative of the function $f(x) = \displaystyle\int_0^{\sqrt{x}} \frac{t^2}{t^2+1} dt$

24. Find the integral:

    (a) $\displaystyle\int_1^2 \left(x + \frac{1}{x}\right)^2 dx$

    (b) $\displaystyle\int_1^2 \frac{x^2+1}{\sqrt{x}} dx$

    (c) $\displaystyle\int_0^{\pi/2} (\cos t + 2\sin t) dt$

    (d) $\int \sqrt[3]{1-x}\, dx$

(e) $\int \frac{(1+\sqrt{x})^9}{\sqrt{x}}\, dx$

(f) $\int \frac{e^x+1}{e^x} dx$

(g) $\int_0^4 \frac{x}{\sqrt{1+2x}} dx$ (HINT: do the substitution $u = 1+2x$)

(h) $\int_1^{1/2} \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$

25. Find the area under the curve $y = \sqrt{x}$ above the x-axis between 0 and 4.

26. A particle moves in a straight line and has acceleration given by $a(t) = t^2 - t$. Its initial velocity is $v(0) = 2$ cm/s and its initial displacement is $s(0) = 1$ cm. Find the position function $s(t)$.

27. Find the vector function $\vec{r}(t)$ that gives the position of a particle at time t having the acceleration $\vec{a}(t) = 2t\vec{\imath} + \vec{\jmath}$, initial velocity $\vec{v}(0) = \vec{\imath} - \vec{\jmath}$, and initial position $(1, 0)$.