Sample problems for Test 2.

1. Find $f'(x)$ for $f(x) = \sqrt{5-x}$ using the definition of derivative.

2. Prove that

   (a) $(cf(x))' = cf'(x)$

   (b) $(f(x) + g(x))' = f'(x) + g'(x)$

   (c) $(x^n)' = nx^{n-1}$

   (d) $(\cos x)' = -\sin x$

   (e) $(\sin x)' = \cos x$

   (f) $(\tan x)' = \sec^2 x$

   (g) $(\sec x)' = \sec x \tan x$

3. Find the derivative $f'(x)$ of each function $f(x)$:

   (a) $f(x) = (x^5 + 3x^3 - x + 10)(x^4 - 2x^2 + 4)$

   (b) $f(x) = \dfrac{x^2 + 3x + 3}{\sqrt{x} + 2}$

   (c) $f(x) = x + x\sqrt[3]{x^2} + x\sqrt{x} + \dfrac{1}{x\sqrt{x}} + \dfrac{x}{\sqrt[3]{x}}$

   (d) $f(x) = \sqrt{\csc 2x} + \tan^2(x^2 + 1)$

   (e) $f(x) = \sqrt{x \cos x + x}$

4. Find the equation of the tangent line to the curve $y = x\sqrt{5-x}$ at the point $(1,2)$.

5. A particle moves on a vertical line so that its coordinate at time t is $y = t^3 - 12t + 3$, $t \geq 0$.

   (a) Find the velocity and acceleration functions.

   (b) When is the particle moving upward?

   (c) Find the distance that particle travels in the time interval $0 \leq t \leq 3$

6. $\lim\limits_{x \to 0} \dfrac{\sin 5x}{\tan 2x}$

7. Find $\frac{dy}{dx}$ if $\sin(x + y) = y^2 \tan x$

8. Find the tangent vector and parametric equations for the line tangent to the curve $\vec{r}(t) = <t^2 + 2t, t^3 - t>$ at the point corresponding to $t = 1$.

9. The vector function $\vec{r}(t) = <t, 25t - 5t^2>$ represents the position of a particle at time t. Find the velocity, speed, and acceleration at $t = 1$.

10. Find y'' if $y = x^2 \cos 3x$

11. Find the 54-th derivative of the function $f(x) = x \cos x$.

12. At what points on the curve $x = t^3 + 4t$, $y = 6t^2$ is the tangent parallel to the line with the equations $x = -7t$, $y = 12t - 5$?

13. A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 0.9 ft/s, how fast is the angle between the ladder and the ground changing when the bottom of the ladder is 8 ft from the wall?

14. A water trough is 10 m long and has a cross-section in the shape of an isosceles trapezoid that is 40 cm wide at the bottom, 100 cm wide at the top, and has height 60 cm. If the trough is being filled with water at the rate of $0.1 m^3/min$ how fast is the water level rising when the water is 40 cm deep?

15. Use differentials to find an approximate value for $(1.97)^6$.

16. Find the linear approximation to $f(x) = \dfrac{1}{1 + x^2}$ at 1.