Math 171-502 Sample Problems for Test 3 Spring 2012

1. If $f(x) = x + x^2 + \mathrm{e}^x$ and $g(x) = f^{-1}(x)$, find $g'(1)$.
2. Find y'' if $y = \mathrm{e}^{-5x}\cos 3x$
3. Solve the equation:
   (a) $\ln(x+6) + \ln(x-3) = \ln 5 + \ln 2$
   (b) $2^x + 3 \cdot 2^x = 24$
4. Find the derivative to the function $f(x) = \ln(\sin^{-1}(x^2)))$
5. Differentiate each function:
   (a) $f(x) = \dfrac{\sqrt[3]{3x-1}\,(x-2)^3}{2\sqrt{x+1}}$
   (b) $f(x) = (x + x^2)^{\tan x}$
6. Find $\cos^{-1}\left(\sin\dfrac{5\pi}{4}\right)$
7. Find the derivative of the function $f(x) = \sin^{-1}(\tan^{-1}(2x^2+3))$
8. Evaluate each limit:
   (a) $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x^2}$
   (b) $\lim\limits_{x\to\infty}(x - \sqrt{x^2-1})$
   (c) $\lim\limits_{x\to 0}\left(\dfrac{1}{x}\right)^{\tan x}$
9. Find the absolute maximum and absolute minimum values of $f(x) = x^3 - 2x^2 + x$ on [-1,1].
10. For the function $y = x^2e^x$ find
    (a) All asymptotes.
    (b) Intervals on which the function is increasing/decreasing.
    (c) All local minima/local maxima, absolute minima/absolute maxima.
    (d) Intervals on which the function is CU/CD.
    (e) Inflection points.
11. A cylindrical can without a top is made to contain V cm^3 of liquid. Find the dimensions that will minimize the cost of the metal to make the can.
12. Find the most general antiderivative of the function
    (a) $f(x) = (\sqrt{x}+1)(x - \sqrt{x} + 1)$

(b) $f(x) = \sin x + \dfrac{2}{1+x^2} + \dfrac{3}{\sqrt{1-x^2}}$

13. A particle is moving with the acceleration $a(t) = t^2 - t$, $s(0) = 0$, $v(0) = 1$. Find the position of the particle.

14. Find the vector-function that describe the position of particle that has an acceleration $\vec{a}(t) = 2t\vec{\imath} + 3\vec{\jmath}$, $\vec{v}(0) = \vec{\imath} - \vec{\jmath}$, and initial position at (1,2).

15. Find the value of the sum $\sum_{i=0}^{5} i(i-1)$.

16. Find the area under the curve $y = x^2 + 3x - 2$ from 1 to 4. Use equal subintervals and take x_i^* to be the right end-point of the i-th interval

17. Express the limit $\lim_{n\to\infty} \dfrac{1}{n} \sum_{i=1}^{n} \dfrac{1}{1+(i/n)^2}$ as a definite integral. Do not evaluate it.

18. Write the expression $\int_{-3}^{5} f(x)dx - \int_{-3}^{0} f(x)dx + \int_{5}^{6} f(x)dx$ as a single integral.