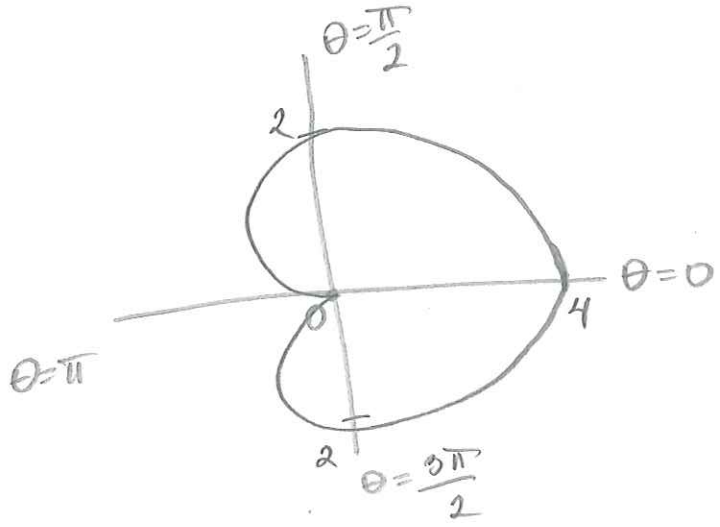
11. (a) Sketch the curve $r = 2(1+\cos\theta)$ in polar coordinates.



$r = 2(1+\cos\theta)$

$r(0) = 2(1+\cos 0) = 4$

$r\left(\frac{\pi}{2}\right) = 2 + 2\cos\frac{\pi}{2} = 2$

$r(\pi) = 2(1+\cos\pi) = 0$

$r\left(\frac{3\pi}{2}\right) = 2$

(b) Find the length of the polar curve $r = 2(1+\cos\theta)$

$L = 2\int_0^{\pi} \sqrt{r^2 + [r']^2}\, d\theta$ | $r' = -2\sin\theta$

$= 2\int_0^{\pi} \sqrt{4(1+\cos\theta)^2 + 4\sin^2\theta}\, d\theta = 2\int_0^{\pi} \sqrt{4(1+2\cos\theta+\cos^2\theta)+4\sin^2\theta}\, d\theta$

$= 2\int_0^{\pi} \sqrt{4 + 8\cos\theta + \underbrace{4\cos^2\theta + 4\sin^2\theta}_{4}}\, d\theta = 2\int_0^{\pi} \sqrt{8+8\cos\theta}\, d\theta$

$= 2\int_0^{\pi} \sqrt{8(1+\cos\theta)}\, d\theta$ | $1+\cos\theta = 2\cos^2\frac{\theta}{2}$

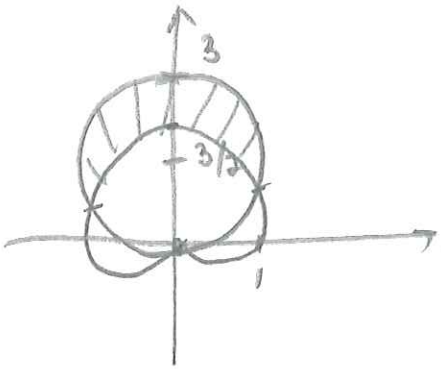
$= 2\int_0^{\pi} \sqrt{16\cos^2\frac{\theta}{2}}\, d\theta = 2\int_0^{\pi} 4\cos\frac{\theta}{2}\, d\theta = 8\cdot 2\sin\frac{\theta}{2}\Big]_0^{\pi}$

$= 16\left(\sin\frac{\pi}{2} - \sin 0\right) = \boxed{16}$

12. A region D lies inside the circle $r = 3\sin\theta$ and outside the cardioid $r = 1 + \sin\theta$.

(a) Sketch the region D.

$r = 3\sin\theta$



$3\sin\theta = 1 + \sin\theta$

$2\sin\theta = 1$

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$.

(b) Find the area of the region D.

$$A = 2 \cdot \frac{1}{2} \int_{\pi/6}^{\pi/2} \left[9\sin^2\theta - (1+\sin\theta)^2 \right] d\theta$$

$$= \int_{\pi/6}^{\pi/2} \left[9\sin^2\theta - 1 - 2\sin\theta - \sin^2\theta \right] d\theta$$

$$= \int_{\pi/6}^{\pi/2} \left[8\sin^2\theta - 1 - 2\sin\theta \right] d\theta =$$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$$= \int_{\pi/6}^{\pi/2} \left[4(1-\cos 2\theta) - 1 - 2\sin\theta \right] d\theta$$

$$= \left[4\theta - 2\sin 2\theta - \theta + 2\cos\theta \right]_{\pi/6}^{\pi/2}$$

$$3\left(\frac{\pi}{2} - \frac{\pi}{6}\right) - 2\left(-\sin\frac{3\pi}{3} + \sin\pi\right) + 2\cos\frac{\pi}{2} - 2\cos\frac{\pi}{6}$$

$$= \pi + 2\sqrt{3} - 2\sqrt{3} = \boxed{\pi}$$