MATH 251, Section ______
Thursday, Sept. 16, 2010

Quiz 3 (Sections 11.5, 11.6, 11.7 (Arc length)).
Dr. M. Vorobets

NAME (print): ______________________________

No credit for unsupported answers will be given. No calculators. Clearly indicate your final answer

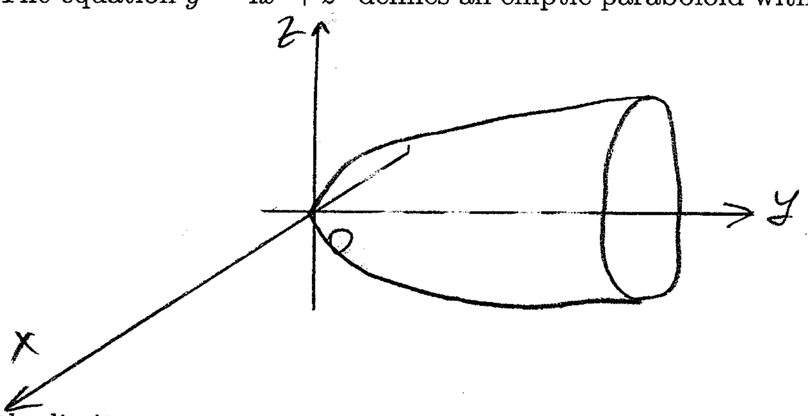
1. *[3 pts.]* Consider the quadric surface $y = 4x^2 + z^2$. Find the traces (write **equations** and the **names** of the curves) of this surface in the planes

   (a) $y = 4$:
   SOLUTION. The equation of the trace of the suface in the plane $y = 4$ is
   $$4 = 4x^2 + z^2 \qquad \text{or} \qquad x^2 + \frac{z^2}{4} = 1.$$
   This equation defines the ellips.

   (b) $x = 0$:
   SOLUTION. The equation of the trace of the surface in the plane $x = 0$ is
   $$y = z^2$$
   and it defines the parabola.

   (c) $z = 0$:
   The equation of the trace of the surface in the plane $z = 0$ is
   $$y = 4x^2$$
   and it defines the parabola.

2. *[2 pts.]* Classify the surface $y = 4x^2 + z^2$ and sketch it.
   SOLUTION. The equation $y = 4x^2 + z^2$ defines an elliptic paraboloid with axis the y-axis



3. *[2 pts.]* Find the limit
$$\lim_{t\to\infty}\left(e^{-t}\,\vec{\imath} + \frac{t-1}{t+1}\,\vec{\jmath} + \tan^{-1} t\,\vec{k}\right)$$
SOLUTION. $\lim_{t\to\infty}\left(e^{-t}\,\vec{\imath} + \frac{t-1}{t+1}\,\vec{\jmath} + \tan^{-1} t\,\vec{k}\right) = \lim_{t\to\infty}\left(e^{-t}\right)\,\vec{\imath} + \lim_{t\to\infty}\left(\frac{t-1}{t+1}\right)\,\vec{\jmath} +$
$\lim_{t\to\infty}\left(\tan^{-1} t\right)\,\vec{k} = 0\vec{\imath} + 1\vec{\jmath} + \frac{\pi}{2}\vec{k} = < 0, 1, \frac{\pi}{2} >$

4. *[3 pts.]* Find the length of the curve given by the vector function

$$\vec{r}(t) = \cos^3 t\, \vec{\imath} + \sin^3 t\, \vec{\jmath} + \cos(2t)\, \vec{k}, \quad 0 \le t \le \frac{\pi}{2}.$$

SOLUTION. $\vec{r'}(t) = -3\cos^2 t \sin t\, \vec{\imath} + 3\sin^2 t \cos t\, \vec{\jmath} - 2\sin(2t)\, \vec{k}$

$|\vec{r'}(t)| = \sqrt{9\cos^4 t \sin^2 t\ + 9\sin^4 t \cos^2 t\ + 4\sin^2(2t)}$

Recall that $\sin(2t) = 2\sin t \cos t$, then $\sin^2(2t) = 4\sin^2 t \cos^2 t$ and

$|\vec{r'}(t)| = \sqrt{\sin^2 t \cos^2 t(9\sin^2 t + 9\cos^2 t + 16)} = \sin t \cos t \sqrt{25} = 5 \sin t \cos t = \frac{5}{2}\sin(2t)$

Then the length of the curve

$$L = \int_0^{\pi/2} \frac{5}{2}\sin(2t)\, dt = -\frac{5}{4}\cos(2t)\Big|_0^{\pi/2} = \frac{5}{2}$$