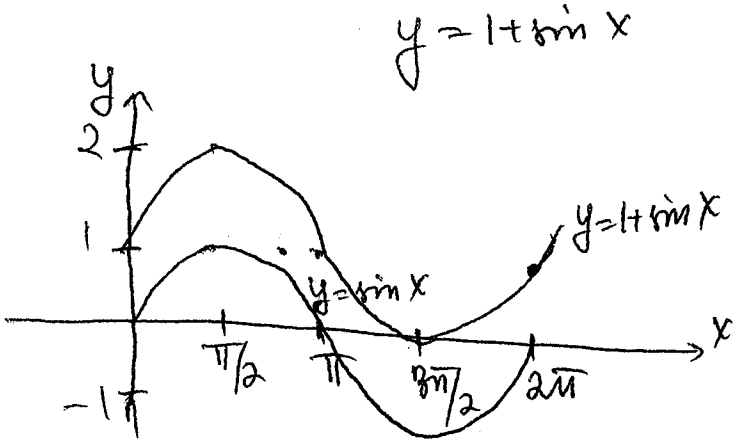
MATH 251, Section _____
Thursday, Oct. 21, 2010

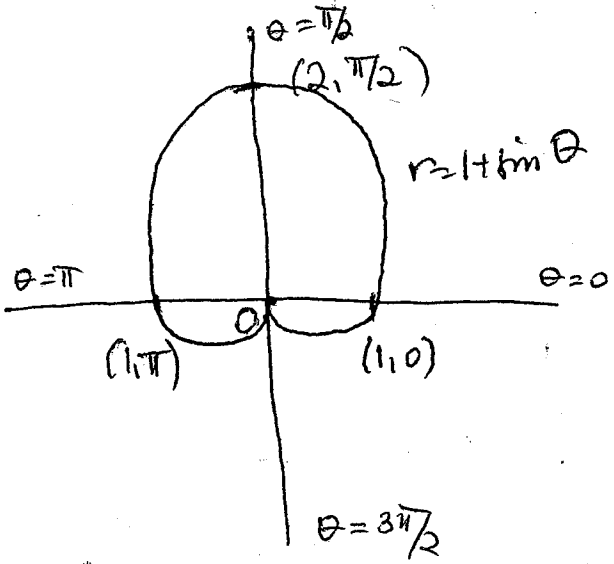
Quiz 8 (Sections 13.4, 13.5).
Dr. M. Vorobets

NAME (print): key

No credit for unsupported answers will be given. Clearly indicate your final answer.

1. *[5 pts.]* Graph the curve $r = 1 + \sin\theta$



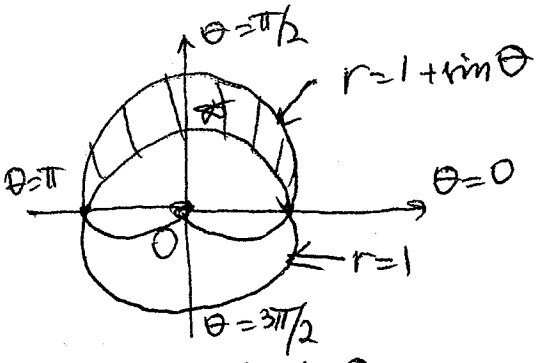


[more problems on back]

2. *[5 pts.]* Evaluate

$$I = \iint_D \sqrt{x^2+y^2}\, dA = \left| \begin{array}{ll} x = r\cos\theta & dA = r\,dr\,d\theta \\ y = r\sin\theta & 0 \le \theta \le \pi \\ \sqrt{x^2+y^2} = r & 1 \le r \le 1+\sin\theta \end{array} \right|$$

where D is the region that lies inside the cardioid $r = 1 + \sin\theta$ and outside the circle $r = 1$.



$0 \le \theta \le \pi$

$1 \le r \le 1+\sin\theta$.

$$I = \int_0^{\pi}\int_1^{1+\sin\theta} r\, r\,dr\,d\theta = \int_0^{\pi}\int_1^{1+\sin\theta} r^2\,dr\,d\theta = \frac{1}{3}\int_0^{\pi} r^3\Big|_{r=1}^{r=1+\sin\theta} d\theta$$

$$= \frac{1}{3}\int_0^{\pi}\left[(1+\sin\theta)^3 - 1\right]d\theta = \frac{1}{3}\int_0^{\pi}\left(1+3\sin\theta+3\sin^2\theta+\sin^3\theta-1\right)d\theta$$

$$= \frac{1}{3}\int_0^{\pi}\left(3\sin\theta+3\sin^2\theta+\sin^3\theta\right)d\theta = \int_0^{\pi}\sin\theta\,d\theta + \int_0^{\pi}\sin^2\theta\,d\theta +$$

$$\frac{1}{3}\int_0^{\pi}\sin^3\theta\,d\theta = -\cos\theta\Big|_0^{\pi} + \frac{1}{2}\int_0^{\pi}(1+\cos 2\theta)\,d\theta + \frac{1}{3}\int_0^{\pi}\sin\theta(1-\cos^2\theta)\,d\theta$$

$$= (-1-1) + \frac{1}{2}\left(\theta+\frac{1}{2}\sin 2\theta\right)\Big|_0^{\pi} + \frac{1}{3}\int_0^{\pi}\sin\theta(1-\cos^2\theta)\,d\theta = \left| \begin{array}{l} u = \cos\theta \\ du = -\sin\theta\,d\theta \\ \theta = 0 \to\ u = 1 \\ \theta = \pi \to\ u = -1 \end{array} \right|$$

$$= 2 + \frac{1}{2}(\pi - 0) - \frac{1}{3}\int_1^{-1}(1-u^2)\,du = 2 + \frac{\pi}{2} + \frac{1}{3}\left(u - \frac{u^3}{3}\right)\Big|_{u=-1}^{u=1}$$

$$= 2 + \frac{\pi}{2} + \frac{1}{3}\left(1-\frac{1}{3}+1-\frac{1}{3}\right) = 2 + \frac{\pi}{2} + \frac{2}{3} - \frac{2}{9} = \boxed{\frac{22}{9} + \frac{\pi}{2}}$$