MATH 251, Section _____
Thursday, Oct. 14, 2010

Quiz 7 (Sections 13.1–13.3).
Dr. M. Vorobets

The quiz is **due Tuesday, Oct. 19 at the beginning of class.**

NAME (print): _____

No credit for unsupported answers will be given. Clearly indicate your final answer

1. *[3 pts.]* Evaluate the double integral

$$\iint_R xe^{xy}\,dA$$

if $R = [0,1] \times [0,1]$

$0 \le x \le 1$
$0 \le y \le 1$

$$\iint_R xe^{xy}\,dA = \int_0^1 \int_0^1 xe^{xy}\,dy\,dx = \int_0^1 x\left(\frac{1}{x}e^{xy}\right)\Big|_{y=0}^{y=1} dx$$

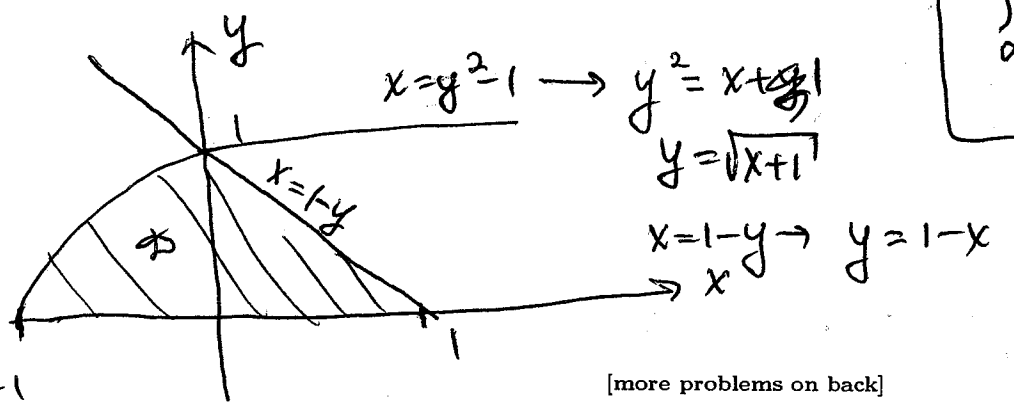
$$= \int_0^1 (e^x - 1)\,dx = (e^x - x)\Big|_0^1 = e^1 - e^0 - 1 = \boxed{e-2}$$

2. *[3 pts.]* Sketch the region of integration and change the order of integration for

$$\int_0^1 \int_{y^2-1}^{1-y} f(x,y)\,dx\,dy$$

$$= \int_{-1}^{0} \int_{0}^{\sqrt{x+1}} f(x,y)\,dy\,dx + \int_{0}^{1} \int_{0}^{1-x} f(x,y)\,dy\,dx$$

D: $x = 1-y$, $y = 0$; $x = y^2 - 1$, $y = 1$



$x = y^2 - 1 \longrightarrow y^2 = x + 1$

$y = \sqrt{x+1}$

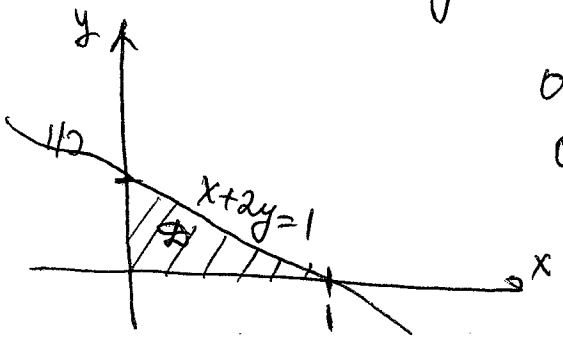
$x = 1 - y \rightarrow y = 1 - x$

[more problems on back]

3. *[4 pts.]* Find the volume of the solid bounded by elliptic paraboloid $z = 2x^2 + y^2 + 1$ and the planes $x + 2y = 1$, $x = 0$, $y = 0$, and $z = 0$.

$$V = \iint_D (2x^2+y^2+1)\,dA$$

D: $x+2y=1$, $x=0$, $y=0$



$0 \le x \le 1$

$0 \le y \le \frac{1-x}{2}$

$$V = \int_0^1 \int_0^{\frac{1-x}{2}} (2x^2+y^2+1)\,dy\,dx$$

$$= \int_0^1 \left(2x^2y + \frac{y^3}{3} + y\right)\Big|_0^{\frac{1-x}{2}} dx$$

$$= \int_0^1 \left(x^2(1-x) + \frac{1}{3}\frac{(1-x)^3}{8} + \frac{1-x}{2}\right) dx$$

$$= \int_0^1 \left(x^2 - x^3 + \frac{1}{24}(1-x)^3 + \frac{1}{2}(1-x)\right) dx$$

$$= \left[\frac{x^3}{3} - \frac{x^4}{4} - \frac{1}{24}\frac{(1-x)^4}{4} - \frac{1}{2}\frac{(1-x)^2}{2}\right]\Big|_0^1 = \frac{1}{3} - \frac{1}{4} + \frac{1}{24}\frac{1}{4} + \frac{1}{2}\frac{1}{2}$$

$$= \boxed{\frac{11}{32}}$$

or $0 \le y \le 1/2$

$0 \le x \le 1-2y$

$$V = \int_0^{1/2} \int_0^{1-2y} (2x^2+y^2+1)\,dx\,dy = \int_0^{1/2} \left(\frac{2}{3}x^3 + y^2x + x\right)\Big|_{x=0}^{x=1-2y} dy$$

$$= \int_0^{1/2} \left[\frac{2}{3}(1-2y)^3 + y^2(1-2y) + 1 - 2y\right] dy$$

$$= \int_0^{1/2} \left[\frac{2}{3}(1-2y)^3 + y^2 - 2y^3 + 1 - 2y\right] dy$$

$$= \left(\frac{2}{3}\left(-\frac{1}{2}\right)\frac{(1-2y)^4}{4} + \frac{y^3}{3} - \frac{2y^4}{4} + y - \frac{2y^2}{2}\right)\Big|_0^{1/2}$$

$$= \frac{1}{8}\left(\frac{1}{3}\right) - \frac{1}{16}\frac{1}{2} + \frac{1}{2} - \frac{1}{4} + \frac{1}{3}\frac{1}{4} = \boxed{\frac{11}{32}}$$