MATH 251, Section ______
Thursday, Nov. 4, 2010

Quiz 10 (Sections 13.9, 13.10, 14.1).
Dr. M. Vorobets

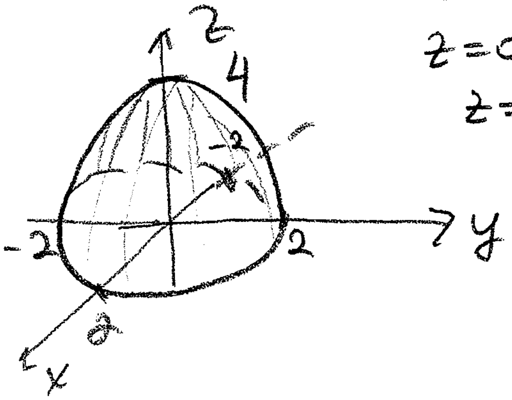
NAME (print): ______________________________

No credit for unsupported answers will be given. Clearly indicate your final answer.

1. *[3 pts.]* Sketch the solid whose volume is given by the integral $\int_0^{2\pi}\int_0^{2}\int_0^{4-r^2} r\,dz\,dr\,d\theta$

$x = r\cos\theta$
$y = r\sin\theta$
$z = z$

$0 \le \theta \le 2\pi$
$0 \le r \le 2$ $\quad r = 2 \Rightarrow x^2+y^2 = 4$ - circle in (xy) plane of radius 2 centered at (0,0)
$0 \le z \le 4 - r^2$
$z = 0$ - (xy) plane.
$z = 4 - r^2 = 4 - x^2 - y^2$ - paraboloid



2. *[2 pts.]* Find the gradient vector field for the function

$$f(x, y, z) = \sqrt{x}\sin(y^2 + z^2)$$

$\nabla f = \langle f_x, f_y, f_z \rangle$

$f_x = \frac{1}{2\sqrt{x}}\sin(y^2+z^2)$

$f_y = \sqrt{x}\cos(y^2+z^2)(2y)$

$f_z = \sqrt{x}\cos(y^2+z^2)(2z)$

$\nabla f = \langle \frac{1}{2\sqrt{x}}\sin(y^2+z^2), \sqrt{x}(2y)\cos(y^2+z^2), \sqrt{x}(2z)\cos(y^2+z^2) \rangle$

[more problems on back]

3. *[5 pts.]* Use spherical coordinates to find the volume of the solid that lies above the cone $z = \sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2 = 2z$.

$$\begin{cases} x = \rho\cos\theta\sin\varphi \\ y = \rho\sin\theta\sin\varphi \\ z = \rho\cos\varphi \end{cases}$$

$z = \sqrt{x^2+y^2}$

$z^2 = x^2+y^2$

$z^2+z^2 = x^2+y^2+z^2$

$2z^2 = x^2+y^2+z^2$

$2\rho^2\cos^2\varphi = \rho^2$

$\cos^2\varphi = \frac{1}{2}$

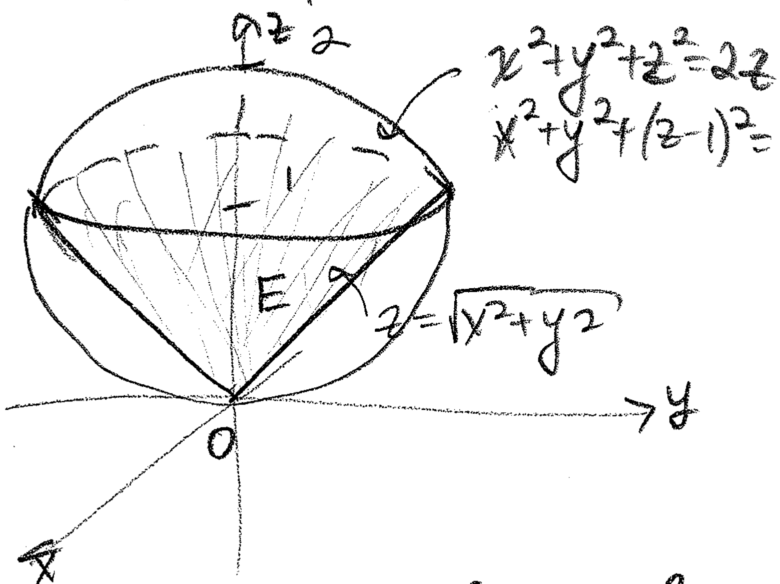
$\cos\varphi = \frac{1}{\sqrt{2}}$

$\varphi = \pi/4.$

$x^2+y^2+z^2 = 2z$

$\rho^2 = 2\rho\cos\varphi$

$\rho = 2\cos\varphi$



$0 \le \rho \le 2\cos\varphi$

$0 \le \varphi \le \frac{\pi}{4}$

$0 \le \theta \le 2\pi$

$$V = \iiint_E dV = \int_0^{2\pi}\int_0^{\pi/4}\int_0^{2\cos\varphi} \rho^2 \sin\varphi \, d\rho\, d\varphi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi/4} \frac{\rho^3}{3}\sin\varphi \Big|_{\rho=0}^{\rho=2\cos\varphi} d\varphi\, d\theta = \frac{1}{3}\int_0^{2\pi}\int_0^{\pi/4} \cos^3\varphi\sin\varphi\, d\theta\, d\varphi$$

$$= \frac{1}{3}\int_0^{2\pi} d\theta \int_0^{\pi/4}\cos^3\varphi\sin\varphi\, d\varphi$$

$$= \frac{2\pi}{3}\int_0^{\pi/4}\cos^3\varphi\sin\varphi\, d\varphi = \left| \begin{array}{l} \cos\varphi = u \\ du = -\sin\varphi\, d\varphi \\ \varphi = 0 \to u = 1 \\ \varphi = \pi/4 \to u = \sqrt{2}/2 \end{array} \right|$$

$$\boxed{\frac{\pi}{2}}$$

$$= \frac{2\pi}{3}\int_1^{\frac{\sqrt{2}}{2}} (-u^3)\, du = -\frac{2\pi}{3}\,\frac{u^4}{4}\Big|_1^{\frac{\sqrt{2}}{2}} = -\frac{2\pi}{3}\left[\left(\frac{\sqrt{2}}{2}\right)^4 - 1\right] = -\frac{2\pi}{3}\left(\frac{4}{16} - 1\right) =$$