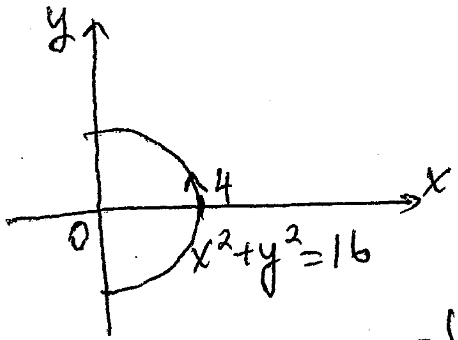
MATH 251, Section ______
Thursday, Nov. 11, 2010

Quiz 11 (Sections 14.2, 14.3).
Dr. M. Vorobets

NAME (print): key

No credit for unsupported answers will be given. Clearly indicate your final answer.

1. *[4 pts.]* Evaluate the line integral $\int_C xy^2 ds$ if C is the right half of the circle $x^2+y^2=16$.



$x = 4\cos t$ $\quad$ $x'(t) = -4\sin t$

$y = 4\sin t$ $\quad$ $y'(t) = 4\cos t$

$-\frac{\pi}{2} \le t \le \frac{\pi}{2}$

$$\int_C xy^2 ds = \int_{-\pi/2}^{\pi/2} 4\cos t \, 16\sin^2 t \sqrt{16\sin^2 t + 16\cos^2 t}\, dt$$

$$= \int_{-\pi/2}^{\pi/2} (4)(16)(4) \cos t \sin^2 t \, dt = \left| \begin{array}{l} u = \sin t \\ du = \cos t\, dt \\ -\pi/2 \to -1 \\ \pi/2 \to 1 \end{array} \right.$$

$$= 256 \int_{-1}^{1} u^2 du = 256 \frac{u^3}{3}\Big|_{-1}^{1} = \cancel{\frac{256}{3}} = 256\frac{2}{3} = \boxed{\frac{512}{3}}$$

2. *[3 pts.]* Find $\int_C \vec{F}\cdot d\vec{r}$ if $\vec{F} = (y+z)\vec{\imath} - x^2\vec{\jmath} - 4y^2\vec{k}$ and C is given by $\vec{r}(t) = t\vec{\imath} + t^2\vec{\jmath} + t^4\vec{k}$, $0 \le t \le 1$.

$\int_C \vec{F}\cdot d\vec{r} = \int_C P dx + Q dy + R dz$ where

$P = y+z,\ Q = -x^2,\ R = -4y^2$

$x(t) = t \quad dx = dt$

$y(t) = t^2 \quad dy = 2t\,dt$

$z(t) = t^4 \quad dz = 4t^3 dt$

$$= \int_C (y+z)dx - x^2 dy - 4y^2 dz = \int_0^1 (t^2+t^4)dt - \int_0^1 t^2(2t)dt - 4\int_0^1 t^4(4t^3)dt$$

$$= \left(\frac{t^3}{3} + \frac{t^5}{5}\right)\Big|_0^1 - 2\frac{t^4}{4}\Big|_0^1 - 16\frac{t^8}{8}\Big|_0^1$$

$$= \frac{1}{3} + \frac{1}{5} - \frac{1}{2} - 2 = \boxed{-\frac{59}{30}}$$

3. *[3 pts.]* Determine whether or not the vector field

$$\vec{F}(x,y) = (x^2+y)\vec{\imath} + (y^2+x)\vec{\jmath}$$

is conservative. If it is, find a function f such that $\vec{F} = \nabla f$.

$P(x,y) = x^2+y$ $\qquad$ $Q(x,y) = y^2+x$

$\frac{\partial P}{\partial y} = 1$ ——— $\frac{\partial Q}{\partial x} = 1$ $-\vec{F}$ is conservative.

f: $\nabla f = \langle f_x, f_y, \cancel{f_z} \rangle = \langle x^2+y, y^2+x \rangle$

$$\begin{cases} f_x = x^2+y \\ f_y = y^2+x \end{cases}$$

$f(x,y) = \int (x^2+y)\,dx = \frac{x^3}{3} + xy + g(y)$

plug f into the 2nd equation: $g(y)$ is unknown

$f_y = x + g'(y) = y^2 + x$

$g'(y) = y^2$

$g(y) = \frac{y^3}{3} + K$, K is a constant.

$$\boxed{f(x,y) = \frac{x^3}{3} + xy + \frac{y^3}{3} + K}$$