

MATH 251, Section _____
Thursday, Dec. 2, 2010

Quiz 13 (Sections 14.6, 14.7).
Dr. M. Vorobets

NAME (print): key

No credit for unsupported answers will be given. Clearly indicate your final answer.

1. [10 pts.] Evaluate $\iint_S x \, dS$, where S is the surface $y = x^2 + 4z$, $0 \leq x \leq 2$, $0 \leq z \leq 2$.

$$S: y = x^2 + 4z$$

parametrization for the
surface S :

$$\begin{cases} x = x \\ y = x^2 + 4z \\ z = z \end{cases} \quad \begin{matrix} 0 \leq x \leq 2 \\ 0 \leq z \leq 2 \end{matrix}$$

$$\iint_S x \, dS = \iint_D x \sqrt{1 + \left[\frac{\partial y}{\partial x}\right]^2 + \left[\frac{\partial y}{\partial z}\right]^2} \, dA$$

$$= \iint_D x \sqrt{1 + [2x]^2 + [4]^2} \, dA = \int_0^2 \int_0^2 x \sqrt{17 + 4x^2} \, dx \, dz$$

$$u = 17 + 4x^2$$

$$du = 8x \, dx$$

$$x=0 \rightarrow u=17$$

$$x=2 \rightarrow u=17+16=33$$

$$= \frac{1}{8} \int_0^2 \int_{17}^{33} \sqrt{u} \, du \, dz = \frac{1}{8} \cdot \int_0^2 \left. \frac{u^{3/2}}{3/2} \right|_{17}^{33} \, dz$$

$$= \frac{1}{8} \cdot \frac{2}{3} (2) (33^{3/2} - 17^{3/2}) = \frac{1}{6} (33^{3/2} - 17^{3/2})$$