MATH 251, Section 506, 507, 508 Quiz#10 (Sections 14.3, 14.4, 14.5).
Thursday, Nov. 7, 2013 Dr. M. Vorobets
**Due Tuesday, Nov. 12, 2013 at the beginning of class.**

NAME (print): key

**No credit for unsupported answers will be given. Clearly indicate your final answer. Staple all the sheets.**

1. Given the vector field $\vec{F}(x,y) = \langle e^{2y}, 1 + 2xe^{2y} \rangle$.

   (a) *[3 pts.]* Find a function $f$ such that $\vec{F} = \nabla f$.

$\nabla f = \langle f_x, f_y \rangle$ 0.5

$$\begin{cases} f_x = e^{2y} & 0.5 \\ f_y = 1 + 2xe^{2y} & 0.5 \end{cases}$$

$\int f_x\,dx = \int e^{2y}\,dx$

0.5 $f(x,y) = xe^{2y} + g(y)$

$$\frac{\partial f}{\partial y} = \cancel{2xe^{2y}} + g'(y) = 1 + \cancel{2xe^{2y}}$$

$g'(y) = 1$

$g(y) = y + C$ 0.5

$$\boxed{f(x,y) = xe^{2y} + y + C}$$ 0.5

   (b) *[1 pts.]* Use part (a) to evaluate $\int_C \vec{F} \cdot d\vec{r}$ if the curve $C$ is given by the vector equation $\vec{r}(t) = \langle te^t, 1+t \rangle, 0 \le t \le 1$.

$\int_C \vec{F} \cdot d\vec{r} = f(e,2) - f(0,1)$

0.25 0.25

$\vec{r}(1) = \langle e, 2 \rangle$

$\vec{r}(0) = \langle 0, 1 \rangle$

$f(e,2) = ee^4 + 2 + C$

$= e^5 + 2 + C$

$f(0,1) = 1 + C$

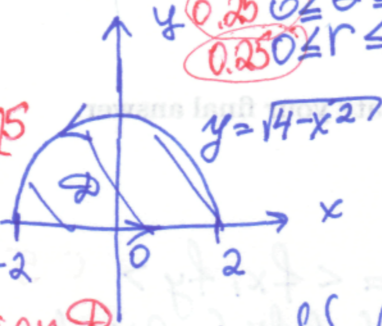
$= ee^4 + 2 + C - 1 - C$

$= \boxed{e^5 + 1}$ 0.5

2. *[3 pts.]* A particle starts at the point (-2,0), moves along the $x$-axis to (2,0), and then along the semicircle $y = \sqrt{4 - x^2}$ to the starting point. Use Green's Theorem to find the work done by the force field $\vec{F}(x, y) = < x, x^3 + 3xy^2 >$.

$0.25 \quad 0 \le \theta \le \pi \quad 0.25$

$0.25 \quad 0 \le r \le 2 \quad 0.25$




0.25 for the region $\mathcal{D}$

$W \overset{0.25}{=} \int_C \vec{F} \cdot d\vec{r} = \int_C x\,dx + (x^3 + 3xy^2)\,dy$

$\overset{0.25}{P(x,y) = x}, \quad \overset{0.25}{Q(x,y) = x^3 + 3xy^2}$

$\overset{0.25}{\frac{\partial P}{\partial y} = 0}, \quad \frac{\partial Q}{\partial x} = \overset{0.25}{3x^2 + 3y^2}$

$W = \iint_{\mathcal{D}} (3x^2 + 3y^2)\,dA = \overset{0.25}{3\int_0^{\pi}\int_0^2 r^3\,dr\,d\theta}$

Polar coord.

$x = r\cos\theta$

$y = r\sin\theta$

$dA = r\,dr\,d\theta$

$= 3(2\pi)\frac{r^4}{4}\Big|_0^2$

$= 6\pi \cdot \frac{16}{4} = \boxed{24\pi} \quad 0.25$

3. *[3 pts.]* Find the curl and the divergence of the vector field

$$\vec{F}(x, y, z) = e^{xyz}\vec{\imath} + \sin(x - y)\vec{\jmath} - \frac{xy}{z}\vec{k}$$

$\operatorname{div}\vec{F} = \overset{0.25}{\frac{\partial}{\partial x}(e^{xyz})} + \overset{0.25}{\frac{\partial}{\partial y}(\sin(x-y))} + \overset{0.25}{\frac{\partial}{\partial z}\left(-\frac{xy}{z}\right)}$

$= \boxed{yze^{xyz} - \cos(x-y) + \frac{xy}{z^2}} \quad 0.25$

$\operatorname{curl}\vec{F} \overset{0.25}{=} \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ e^{xyz} & \sin(x-y) & -\frac{xy}{z} \end{vmatrix}$

$= \vec{\imath}\left(\overset{0.25}{\frac{\partial}{\partial y}\left(-\frac{xy}{z}\right)} - \overset{0.25}{\frac{\partial}{\partial z}(\sin(x-y))}\right)$

$-\vec{\jmath}\left(\overset{0.25}{\frac{\partial}{\partial x}\left(-\frac{xy}{z}\right)} - \overset{0.25}{\frac{\partial}{\partial z}(e^{xyz})}\right)$

$+\vec{k}\left(\overset{0.25}{\frac{\partial}{\partial x}(\sin(x-y))} - \overset{0.25}{\frac{\partial}{\partial y}(e^{xyz})}\right)$

$\underset{0.25}{=} \boxed{-\frac{x}{z}\vec{\imath} + \left(\frac{y}{z} + xye^{xyz}\right)\vec{\jmath} + \left(\cos(x-y) - xze^{xyz}\right)\vec{k}}$