MATH 251, Section 506,507,508 Quiz#1 (Sections 11.1 – 11.4)
Thursday, Sept. 5, 2013 Dr. M. Vorobets

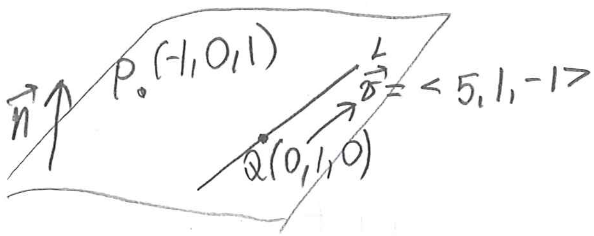
Due Tuesday, Sept. 10, 2013 at the beginning of class.

NAME (print): KEY

No credit for unsupported answers will be given. Clearly indicate your final answer

1. *[3 pts.]* Find an equation of the plane that passes through the point $(-1, 0, 1)$ and contains the line $x = 5t$, $y = 1 + t$, $z = -t$.

$L \parallel \vec{v} = <5, 1, -1>$, $Q(0,1,0)$ lies in the line



$\vec{PQ} = <1,1,-1>$ 1pt

$\vec{n} = \vec{PQ}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 5 & 1 & -1 \\ 1 & 1 & -1 \end{vmatrix}$

$= \vec{i}\begin{vmatrix} 1 & -1 \\ 1 & -1 \end{vmatrix} - \vec{j}\begin{vmatrix} 5 & -1 \\ 1 & -1 \end{vmatrix} + \vec{k}\begin{vmatrix} 5 & 1 \\ 1 & 1 \end{vmatrix}$

$= 0\cdot\vec{i} + 4\vec{j} + 4\vec{k} = <0,4,4>$ 1 pt

Equation: 1pt $\boxed{4(y)+4(z-1)=0}$ OR $\boxed{y+z-1=0}$

2. (a) *[2 pts.]* Find an equation of the plane π that passes through the point $P(2, 8, 5)$ and is orthogonal to the line L given by $x = 2 + t$, $y = 2 - 3t$, $z = 5t$.

L is $\parallel$ to $\vec{v} = <1,-3,5>$

The plane is $\perp$ to $\vec{v}$ — 1pt

Equation: 1pt $\boxed{1(x-2) - 3(y-8) + 5(z-5) = 0}$

OR $\boxed{x - 3y + 5z - 3 = 0}$

(b) *[1 pts.]* Find the point S of intersection of the plane π and the line L.

$2 + t - 3(2-3t) + 5(5t) - 3 = 0$

$35t = 7$, $\boxed{t = 1/5}$ 0.5 pt.

$x = \frac{11}{5}$, $y = \frac{7}{5}$, $z = 1$

$\boxed{S\left(\frac{11}{5}, \frac{7}{5}, 1\right)}$ 0.5 pt.

(c) *[1 pts.]* Find $|PS|$, that is, the distance from the point P to the line L.

$|PS| = \sqrt{\left(\frac{11}{5}-2\right)^2 + \left(\frac{7}{5}-8\right)^2 + (1-5)^2}$

$= \boxed{\sqrt{\frac{1490}{25}} \approx 7.72}$ 1pt

[more problems on back]

$\sqrt{\frac{298}{5}}$

3. *[3 pts.]* Find **parametric** equations of a line that passes through the point $(1, 1, 1)$ and is parallel to the line of intersection of the planes $x + z = 1$ and $y + z = 1$.

$x+z=1$ is $\perp$ to $\vec{n}_1 = \langle 1, 0, 1 \rangle$ 0.5 pt

$y+z=1$ is $\perp$ to $\vec{n}_2 = \langle 0, 1, 1 \rangle$ 0.5 pt

The line is parallel to the vector

$$\vec{v} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix}$$

$$= \vec{i} \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} - \vec{j} \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} + \vec{k} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$= -\vec{i} - \vec{j} + \vec{k} = \langle -1, -1, 1 \rangle.$$ 1 pt

Equations:

$$\boxed{\begin{aligned} x &= 1 - t \\ y &= 1 - t \\ z &= 1 + t \end{aligned}}$$

1 pt