MATH 251, Section 506, 507, 508
Thursday, Sept. 19, 2013

Quiz 3 (Sections 11.6, 11.7, 12.1).
Dr. M. Vorobets

NAME (print): KEY

No credit for unsupported answers will be given. Calculators can be used for simple arithmetic operations only! Clearly indicate your final answer

1. *[3 pts.]* Find $\vec{r}(t)$ if $\vec{r}\,'(t) = t^2\vec{\imath} + \cos(t)\vec{\jmath} - te^t\vec{k}$ and $\vec{r}(0) = \vec{\imath} - 2\vec{\jmath} + \vec{k}$.

$\vec{r}(t) = \langle \int t^2\,dt, \int \cos t\,dt, -\int te^t\,dt \rangle$

$= \langle \frac{t^3}{3} + C_1, \sin t + C_2, -te^t + e^t + C_3 \rangle$ 0.25pts 0.25pts 1pt

$\int te^t\,dt = \int t\,d(e^t) = te^t - \int e^t\,dt = te^t - e^t + C_3$

$\vec{r}(0) = \langle C_1, C_2, 1 + C_3 \rangle = \langle 1, -2, 1 \rangle$

$C_1 = 1$ 0.5pts, $C_2 = -2$ 0.5pts, $C_3 = 0$ 0.5 pts.

$$\boxed{\vec{r}(t) = \langle \frac{t^3}{3} + 1, \sin t - 2, -te^t + e^t \rangle}$$

2. *[4 pts.]* Find the curvature of the curve $\vec{r}(t) = \langle 1 + t, 1 - t, 3t^2 \rangle$.

$$K(t) = \frac{|\vec{r}\,' \times \vec{r}\,''|}{|\vec{r}\,'|^3}$$ 0.5pt.

$\vec{r}\,'(t) = \langle 1, -1, 6t \rangle$ 0.5pt., $|\vec{r}\,'(t)| = \sqrt{2 + 36t^2}$ 0.5pt.

$\vec{r}\,''(t) = \langle 0, 0, 6 \rangle$ 0.5pt

$$\vec{r}\,' \times \vec{r}\,'' = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 1 & -1 & 6t \\ 0 & 0 & 6 \end{vmatrix} = \vec{\imath}\begin{vmatrix} -1 & 6t \\ 0 & 6 \end{vmatrix} - \vec{\jmath}\begin{vmatrix} 1 & 6t \\ 0 & 6 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & -1 \\ 0 & 0 \end{vmatrix}$$

$= -6\vec{\imath} - 6\vec{\jmath} = \langle -6, -6, 0 \rangle$ 1pt.

$|\vec{r}\,' \times \vec{r}\,''| = \sqrt{72}$ 0.5

0.5pt. $K(t) = \dfrac{\sqrt{72}}{(2 + 36t^2)^{3/2}} = \boxed{\dfrac{6\sqrt{2}}{(2 + 36t^2)^{3/2}}}$

[more problems on back]

3. *[3 pts.]* Sketch the domain of the function

$$f(x,y) = \sqrt{x^2+y^2-1} + \ln(4-x^2-y^2)$$

$\sqrt{x^2+y^2-1}$ is defined if $x^2+y^2-1 \geq 0$

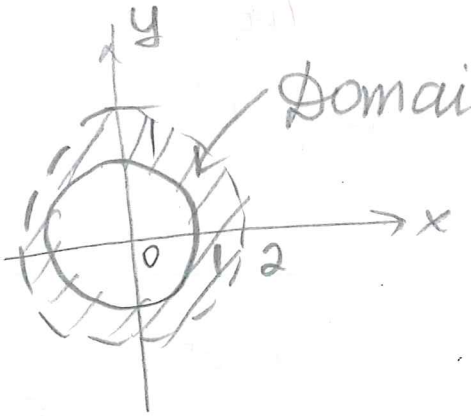
or $x^2+y^2 \geq 1$

point that lie on the circle and outsid of the circle $x^2+y^2=1$.

$\ln(4-x^2-y^2)$ is defined if $4-x^2-y^2 > 0$

or $x^2+y^2 < 4$

(points that lie inside the circle $x^2+y^2=4$)



1pt for the dotted circle

1pt for the another circle

1pt for making the space between the circles.