

MATH 251, Section 506, 507, 508
Thursday, Sept. 27, 2013

Quiz 4 (Sections 12.3, 12.4).
Dr. M. Vorobets

NAME (print): key

No credit for unsupported answers will be given. Calculators can be used for simple arithmetic operations only! Clearly indicate your final answer

1. [5 pts.] Find all second partial derivatives of the function $f(x, y) = x^3 \ln(x - y)$.

$$\frac{\partial f}{\partial x} = 3x^2 \ln(x-y) + \frac{x^3}{x-y}, \quad \frac{\partial f}{\partial y} = -\frac{x^3}{x-y} = -x^3(x-y)^{-1}$$

$$\frac{\partial^2 f}{\partial x^2} = 6x \ln(x-y) + \frac{3x^2}{x-y} + \frac{3x^2(x-y) - x^3}{(x-y)^2}$$

$$= 6x \ln(x-y) + \frac{3x^2}{x-y} + \frac{2x^3 - 3x^2y}{(x-y)^2}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{-3x^2}{x-y} - \frac{x^3}{(x-y)^2}$$

$$\frac{\partial^2 f}{\partial y^2} = -(-1)x^3(x-y)^{-2}(-1) = -\frac{x^3}{(x-y)^2}$$

2. [5 pts.] Use differentials to approximate the number $\sqrt{0.99}e^{0.02}$

$$f(a+\Delta x, b+\Delta y) \approx f(a, b) + f_x(a, b)\Delta x + f_y(a, b)\Delta y$$

$$f(x, y) = \sqrt{x} e^y \quad \text{0.25 pt} \quad \text{0.25 pt}$$

$$a + \Delta x = 0.99$$

$$a = 1, \Delta x = -0.01$$

$$b + \Delta y = 0.02$$

$$b = 0, \Delta y = 0.02$$

$$f(x, y) = \sqrt{x} e^y \quad | \quad f(1, 0) = 1 \quad \text{0.25 pt} \quad \text{0.5 pt}$$

$$0.5 \text{ pt} \quad \frac{\partial f}{\partial x} = \frac{1}{2} x^{-1/2} e^y \quad | \quad \frac{\partial f}{\partial x}(1, 0) = \frac{1}{2} \quad \text{0.25 pt} \quad \text{0.5 pt}$$

$$0.5 \text{ pt} \quad \frac{\partial f}{\partial y} = \sqrt{x} e^y \quad | \quad \frac{\partial f}{\partial y}(1, 0) = 1 \quad \text{0.25 pt} \quad \text{0.5 pt}$$

$$\sqrt{0.99} e^{0.02} \approx \boxed{1 - \frac{1}{2}(0.01) + 0.02} \quad \text{1 pt}$$