MATH 251, Section 506, 507, 508 Quiz#5 (Sections 12.5, 12.6)
Thursday, Sept. 26, 2013 Dr. M. Vorobets

Due Tuesday, Oct. 1, 2013 at the beginning of class.

NAME (print): KEY

No credit for unsupported answers will be given. Clearly indicate your final answer

1. *[3 pts.]* Let $z = x\tan^{-1}(xy)$, where $x = ts$ and $y = se^t$. Find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$.

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

$$\frac{\partial z}{\partial x} = \tan^{-1}(xy) + \frac{xy}{1+x^2y^2} \quad 0.5$$

$$\frac{\partial z}{\partial y} = \frac{x^2}{1+x^2y^2} \quad 0.5$$

$$\frac{\partial x}{\partial s} = t \ (0.25), \quad \frac{\partial x}{\partial t} = s \ (0.25)$$

$$\frac{\partial y}{\partial s} = e^t \ (0.25), \quad \frac{\partial y}{\partial t} = se^t \ (0.25)$$

$$0.5 \quad \boxed{\frac{\partial z}{\partial s} = \left(\tan^{-1}(xy) + \frac{xy}{1+x^2y^2}\right)t + \frac{x^2}{1+x^2y^2}e^t}$$

$$0.5 \quad \boxed{\frac{\partial z}{\partial t} = \left(\tan^{-1}(xy) + \frac{xy}{1+x^2y^2}\right)s + \frac{x^2}{1+x^2y^2}se^t}$$

2. *[3 pts.]* If $y^2ze^{x+y} - \sin(xyz) = 0$, find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

$$F(x,y,z) = y^2ze^{x+y} - \sin(xyz) \quad 0.5$$

$$\frac{\partial F}{\partial x} = y^2ze^{x+y} - \cos(xyz)(yz) \quad 0.5$$

$$\frac{\partial F}{\partial y} = 2yze^{x+y} + y^2ze^{x+y} - \cos(xyz)(xz) \quad 0.5$$

$$\frac{\partial F}{\partial z} = y^2e^{x+y} - \cos(xyz)(xy) \quad 0.5$$

$$\boxed{\frac{\partial z}{\partial x} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}} = -\frac{y^2ze^{x+y} - yz\cos(xyz)}{y^2e^{x+y} - xy\cos(xyz)}} \quad 0.5$$

[more problems on back]

$$0.5 \quad \boxed{\frac{\partial z}{\partial y} = -\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}} = -\frac{2yze^{x+y} + y^2ze^{x+y} - xz\cos(xyz)}{y^2e^{x+y} - xy\cos(xyz)}}$$

3. *[2 pts.]* Find the directional derivative of $f(x,y) = 2\sqrt{x} - y^2$ in the direction of $\vec{v} = < 3, -4 >$.

$$|\vec{v}| = \sqrt{9+16} = \sqrt{25} = 5 \quad 0.25$$

$$\vec{u} = \frac{\vec{v}}{|\vec{v}|} = < \frac{3}{5}, -\frac{4}{5} > 0.25$$

$$\nabla f = < 2\tfrac{1}{2}x^{-1/2}, -2y > = < \overset{0.25}{x^{-1/2}}, \overset{0.25}{-2y} >$$

$$D_{\vec{u}} f \overset{0.5}{=} \nabla f \cdot \vec{u} = < x^{-1/2}, -2y > \cdot < \frac{3}{5}, -\frac{4}{5} >$$

$$= \boxed{\frac{3}{5}x^{-1/2} - \frac{8}{5}y} \quad 0.5$$

4. *[2 pts.]* Find an equation of the tangent plane to $x^2 - 2y^2 - 3z^2 + xyz = 4$ at the point $(3, -2, -1)$.

$$F(x,y,z) = x^2 - 2y^2 - 3z^2 + xyz - 4 \quad 0.25$$

$$\nabla F = < \overset{0.25}{2x + yz}, \overset{0.25}{-4y + xz}, \overset{0.25}{-6z + xy} >$$

$$\nabla F(3,-2,-1) = < 6+2, 8-3, 6-6 >$$

$$= < 8, 5, 0 > \quad 0.5$$

An equation of the tangent plane is

$$\boxed{8(x-3) + 5(y+2) + 0(z+1) = 0} \quad 0.5$$

OR

$$\boxed{8x + 5y - 14 = 0}$$