MATH 251, Section 506, 507, 508 Thursday, Oct. 16, 2013

Quiz#6 (Sections 13.2, 13.3) Dr. M. Vorobets

NAME (print): key

No credit for unsupported answers will be given. Clearly indicate your final answer

1. *[3 pts.]* Evaluate the double integral

$$\iint_R xe^{xy}dA = \int_0^1 \int_0^1 xe^{xy}\,dy\,dx$$ 1pt.

if $R = [0,1] \times [0,1]$

$$= \int_0^1 x e^{xy} \frac{1}{x} \Big]_{y=0}^{y=1} dx$$ 1pt.

$$= \int_0^1 e^x dx - \int_0^1 dx$$

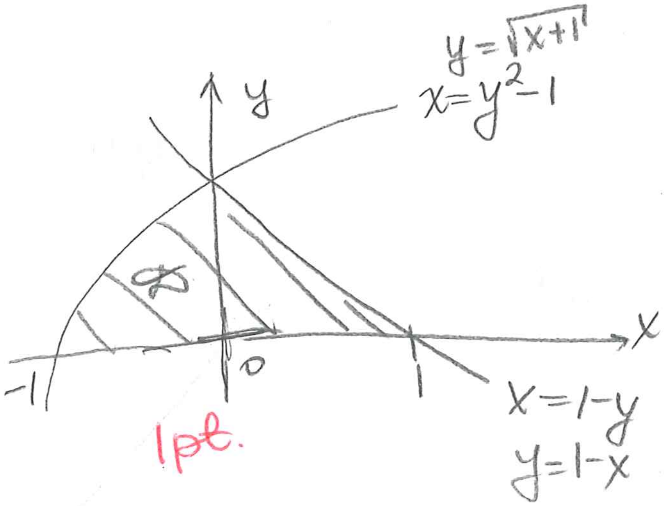
$$= e^x \Big]_0^1 - x \Big]_0^1$$

$$= e - 1 - 1$$

$$= \boxed{e-2}$$ 1pt.

2. *[3 pts.]* Sketch the region of integration and change the order of integration for

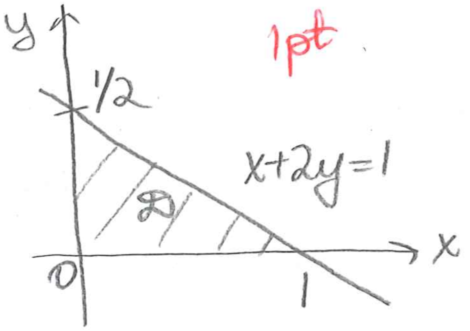
$$\int_0^1 \int_{y^2-1}^{1-y} f(x,y)\,dxdy$$



$$= \boxed{\int_{-1}^{0} \int_{0}^{\sqrt{x+1}} f(x,y)\,dydx + \int_{0}^{1} \int_{0}^{1-x} f(x,y)\,dydx}$$

0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25

3. *[4 pts.]* Find the volume of the solid bounded by elliptic paraboloid $z = 2x^2 + y^2 + 1$ and the planes $x + 2y = 1$, $x = 0$, $y = 0$, and $z = 0$.



1pt

$0 \le x \le 1$

$0 \le y \le \frac{1-x}{2}$

$$V = \iint_D (2x^2+y^2+1)\,dA$$

$$= \int_0^1 \int_0^{\frac{1-x}{2}} (2x^2+y^2+1)\,dy\,dx \quad \text{1pt.}$$

$$= \int_0^1 \left[(2x^2+1)y + \frac{y^3}{3}\right]_{y=0}^{y=\frac{1-x}{2}} dx \quad \text{1pt.}$$

$$= \int_0^1 \left[(2x^2+1)\frac{1-x}{2} + \frac{1}{3}\frac{(1-x)^3}{8}\right] dx$$

$$= \int_0^1 \left[x^2 - x^3 + \frac{1}{2} - \frac{x}{2} + \frac{1}{24}(1-x)^3\right] dx$$

$$= \left[\frac{x^3}{3} - \frac{x^4}{4} + \frac{1}{2}x - \frac{x^2}{4} - \frac{1}{24}\frac{(1-x)^4}{4}\right]_{x=0}^{x=1}$$

$$= \frac{1}{3} - \frac{1}{4} + \frac{1}{2} - \frac{1}{4} + \frac{1}{96} = \frac{33}{96} = \boxed{\frac{11}{32}}$$

1pt.

OR

$0 \le y \le 1/2$

$0 \le x \le 1-2y$

$$V = \iint_D (2x^2+y^2+1)\,dA$$

$$= \int_0^{1/2} \int_0^{1-2y} (2x^2+y^2+1)\,dx\,dy \quad \text{1pt.}$$

$$= \int_0^{1/2} \left(2\frac{x^3}{3} + y^2x + x\right)_{x=0}^{x=1-2y} dy \quad \text{1pt.}$$

$$= \int_0^{1/2} \left(\frac{2}{3}(1-2y)^3 + y^2(1-2y) + 1 - 2y\right) dy$$

$$= \int_0^{1/2} \left(\frac{2}{3}(1-2y)^3 + y^2 - 2y^3 + 1 - 2y\right) dy$$

$$= \left[\frac{2}{3}\left(-\frac{1}{2}\right)\frac{(1-2y)^4}{4} + \frac{y^3}{3} - \frac{2y^4}{4} + y - \frac{2y^2}{2}\right]_0^{1/2}$$

1pt (if the limits are changed)

$$= \boxed{\frac{11}{32}}$$