MATH 251, Section 506, 507, 508 Thursday, Oct. 24, 2013

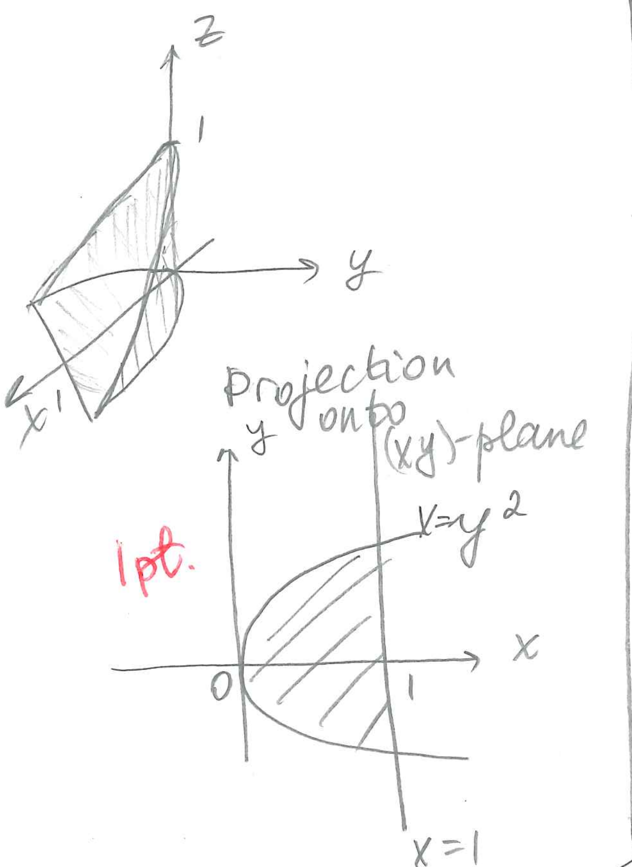
Quiz#8 (Sections 13.8, 13.10) Dr. M. Vorobets

Due Thursday, Oct. 31, 2013 at the beginning of class.

NAME (print): key

No credit for unsupported answers will be given. Clearly indicate your final answer. Staple all the sheets.

1. *[6 pts.]* Evaluate $\iiint_E (x+2y)dV$ if E is bounded by the cylinder $x = y^2$ and the planes $z = 0$ and $x + z = 1$.



0.5 0.5
$0 \le z \le 1-x$

0.5 0.5
$-1 \le y \le 1$

0.5 0.5
$y^2 \le x \le 1$

$$\iiint_E (x+2y)dV$$

0.5

$$= \int_{-1}^{1}\int_{y^2}^{1}\int_{0}^{1-x} (x+2y)\,dz\,dx\,dy$$

$$= \int_{-1}^{1}\int_{y^2}^{1} (x+2y)z\Big]_{z=0}^{z=1-x} dx\,dy$$

0.5

$$= \int_{-1}^{1}\int_{y^2}^{1} (x+2y)(1-x)\,dx\,dy$$

$$= \int_{-1}^{1}\int_{y^2}^{1} (x-x^2+2y-2xy)\,dx\,dy = \int_{-1}^{1}\left[\frac{x^2}{2}-\frac{x^3}{3}+2xy-x^2y\right]_{x=y^2}^{x=1} dy$$

0.5

$$= \int_{-1}^{1}\left[\frac{1}{2}-\frac{1}{3}+2y-y-\frac{y^4}{2}+\frac{y^6}{3}-2y^3+y^5\right]dy$$

$$= \left[\frac{y}{6}+\frac{y^2}{2}-\frac{y^5}{10}+\frac{y^7}{21}-\frac{2y^4}{4}+\frac{y^6}{6}\right]_{-1}^{1}$$

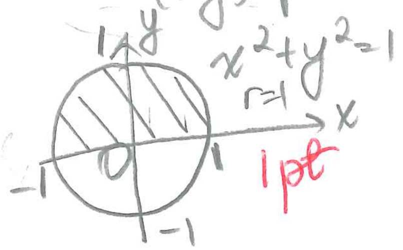
0.5

$$= \frac{2}{2\cdot 3}-\frac{2}{10}+\frac{2}{21} = \frac{1}{3}-\frac{1}{5}+\frac{2}{21} = \boxed{\frac{8}{35}}$$

2. *[7 pts.]* Use cylindrical coordinates to evaluate $\iiint_E xz\,dV$, where E is bounded by the planes $z = 0$, $z = y$, and the cylinder $x^2 + y^2 = 1$ in the half-space $y \geq 0$.

$0 \le z \le y$

projection onto the (xy)-plane



cylindrical coord:

$$\left.\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \\ z &= z \\ dV &= r\,dz\,dr\,d\theta \end{aligned}\right] \text{1 pt}$$

0.5 $\quad 0 \le z \le r\sin\theta \quad$ 0.5

0.5 $\quad 0 \le r \le 1 \quad$ 0.5

0.5 $\quad 0 \le \theta \le \pi \quad$ 0.5

0.5 pt

$$\iiint_E xz\,dV = \int_0^\pi \int_0^1 \int_0^{r\sin\theta} r\cos\theta\, z\, r\,dz\,dr\,d\theta$$

$$= \int_0^\pi \int_0^1 \int_0^{r\sin\theta} r^2 z \cos\theta\,dz\,dr\,d\theta$$

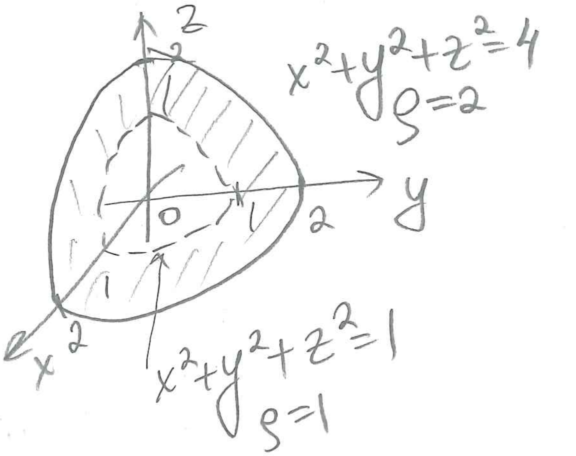
0.5 $\qquad$ 0.5

$$= \int_0^\pi \int_0^1 r^2 \left.\frac{z^2}{2}\right]_{z=0}^{z=r\sin\theta} \cos\theta\,dr\,d\theta = \frac{1}{2}\int_0^\pi \int_0^1 r^4 \sin^2\theta\cos\theta\,dr\,d\theta$$

$$= \frac{1}{2}\int_0^\pi \cos\theta\sin^2\theta\,d\theta \int_0^1 r^4\,dr \quad \left| \begin{aligned} u &= \sin\theta \\ du &= \cos\theta\,d\theta \end{aligned};\ \begin{aligned} \theta = 0 &\to u = 0 \\ \theta = \pi &\to u = 0 \end{aligned} \right|$$

$$= \frac{1}{2}\int_0^0 u^2\,du \int_0^1 r^4\,dr = \boxed{0} \quad 0.5$$

3. *[7 pts.]* Use spherical coordinates to evaluate $\iiint_E xe^{(x^2+y^2+z^2)^2}\,dV$, where E is the solid that lies between the spheres $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=4$ in the first octant.



$$\left.\begin{aligned} x &= \rho\cos\theta\sin\varphi \\ y &= \rho\sin\theta\sin\varphi \\ z &= \rho\cos\varphi \\ dV &= \rho^2\sin\varphi\, d\rho\, d\varphi\, d\theta \end{aligned}\right\} \text{1 pt}$$

$x^2+y^2+z^2=\rho^2$

0.5 $1 \le \rho \le 2$ 0.5

0.5 $0 \le \varphi \le \pi/2$ 0.5

0.5 $0 \le \theta \le \pi/2$ 0.5

1 pt

$$\iiint_E xe^{(x^2+y^2+z^2)^2}\,dV = \int_0^{\pi/2}\int_0^{\pi/2}\int_1^2 \rho\cos\theta\sin\varphi\, e^{\rho^4}\rho^2\sin\varphi\, d\rho\, d\theta\, d\varphi$$

$$= \int_0^{\pi/2}\int_0^{\pi/2}\int_1^2 \cos\theta\sin^2\varphi\,\rho^3 e^{\rho^4}\,d\rho\, d\varphi\, d\theta$$

$$= \int_0^{\pi/2}\cos\theta\, d\theta \int_0^{\pi/2}\sin^2\varphi\, d\varphi \int_1^2 \rho^3 e^{\rho^4}\, d\rho$$

$u=\rho^4$

$du = 4\rho^3 d\rho$

$1 \to 1$

$2 \to 2^4 = 16$

$$= \frac{1}{4}\int_0^{\pi/2}\cos\theta\, d\theta \int_0^{\pi/2}\frac{1-\cos 2\varphi}{2}\, d\varphi \int_1^{16} e^u\, du$$

0.5 0.5 0.5

$$= \frac{1}{8}\sin\theta\Big|_0^{\pi/2}\cdot\left(\varphi - \frac{1}{2}\sin 2\varphi\right)\Big|_0^{\pi/2}\cdot e^u\Big|_1^{16}$$

$$= \frac{1}{8}\cdot\frac{\pi}{2}\left(e^{16}-e\right)$$

$$= \boxed{\frac{\pi}{16}\left(e^{16}-e\right)}$$ 0.5