MATH 251, Section 506,507,508
Thursday, Nov. 7, 2013

Quiz 9 (Sections 14.1, 14.2).
Dr. M. Vorobets

NAME (print): key

No credit for unsupported answers will be given. Clearly indicate your final answer.

1. *[2 pts.]* Find the gradient vector field for the function

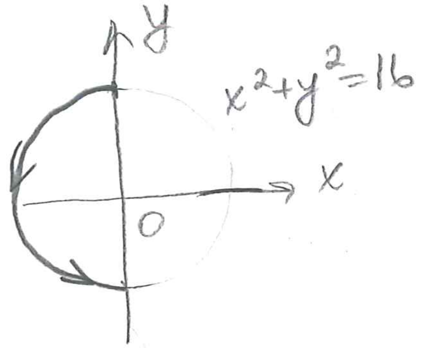
$$f(x, y, z) = \sqrt{x}\sin(y^2 + z^2)$$

$$\nabla f = \langle f_x, f_y, f_z \rangle \quad 0.5$$

$$= \langle \tfrac{1}{2}x^{-1/2}\sin(y^2+z^2), \sqrt{x}\cos(y^2+z^2)(2y), \sqrt{x}\cos(y^2+z^2)2z \rangle$$

0.5, 0.5, 0.5

2. *[4 pts.]* Evaluate the line integral $\int_C xy^2 ds$ if C is the left half of the circle $x^2 + y^2 = 16$.



$x = 4\cos t$ 0.5, $x'(t) = -4\sin t$ 0.25

$y = 4\sin t$ 0.5, $y'(t) = 4\cos t$ 0.25

0.5 $\frac{\pi}{2} \le t \le \frac{3\pi}{2}$ 0.5

$$ds = \sqrt{[x']^2 + [y']^2}\,dt$$

$$= \sqrt{16\sin^2 t + 16\cos^2 t}\,dt$$

$$= 4\,dt \quad 0.5$$

$$\int_C xy^2 ds = \int_{\pi/2}^{3\pi/2} 4\cos t \; 16\sin^2 t \; 4\,dt \quad 0.5$$

$$= 256\int_{\pi/2}^{3\pi/2} \cos t \sin^2 t\,dt$$

0.25

$u = \sin t$
$du = \cos t\,dt$
$t = \frac{\pi}{2} \to u = 1$
$t = \frac{3\pi}{2} \to u = -1$

$$= 256\int_1^{-1} u^2\,du \quad 0.25$$

$$= 256\,\frac{u^3}{3}\Big]_1^{-1} = \boxed{-\frac{512}{3}}$$

3. *[4 pts.]* Find $\int_C \vec{F} \cdot d\vec{r}$ if $\vec{F} = (y+z)\vec{\imath} - x^2\vec{\jmath} - 4y^2\vec{k}$ and C is given by $\vec{r}(t) = t\vec{\imath} - t^3\vec{\jmath} + 2t^2\vec{k}$, $0 \le t \le 1$.

$$\vec{F} = \langle y+z, -x^2, -4y^2 \rangle$$

$$\vec{r} = \langle t, -t^3, 2t^2 \rangle$$

0.25 0.25 0.25

$$\vec{F}(\vec{r}(t)) = \langle -t^3+2t^2, -t^2, -4t^6 \rangle$$

0.25 0.25 0.25

$$\vec{r}\,'(t) = \langle 1, -3t^2, 4t \rangle$$

0.25

$$\int_C \vec{F} \cdot d\vec{r} = \int_0^1 F(\vec{r}(t)) \cdot \vec{r}\,'(t)\, dt$$

0.25 0.25 0.25 0.25

$$= \int_0^1 (-t^3 + 2t^2 + 3t^4 - 16t^7)\, dt$$

0.25 0.25 0.25 0.25

$$= \left[-\frac{t^4}{4} + \frac{2t^3}{3} + \frac{3t^5}{5} - \frac{16t^8}{8} \right]_0^1$$

$$= -\frac{1}{4} + \frac{2}{3} + \frac{3}{5} - 2$$ 0.25

$$= \boxed{-\frac{59}{60}}$$