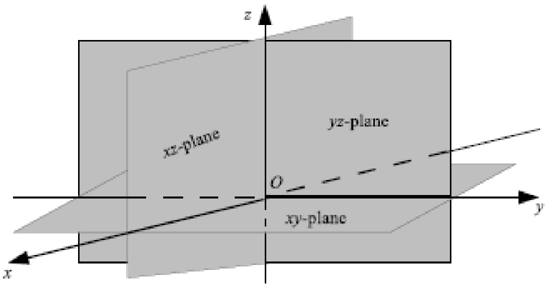
Chapter 11. **Three-dimensional analytic geometry and vectors**
Section 11.1 **Three-dimensional coordinate system**

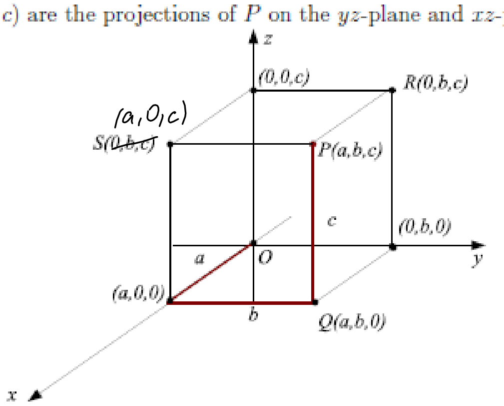
In order to represent points in space, we first choose a fixed point O (the origin) and tree directed lines through O that are perpendicular to each other, called the **coordinate axes** and labeled the x-axis, y-axis, and z-axis. Usually we think of the x and y-axes as being horizontal and z-axis as being vertical.

The direction of z-axis is determined by the **right-hand rule**: if your index finger points in the positive direction of the x-axis, middle finger points in the positive direction of the y-axis, then your thumb points in the positive direction of the z-axis.

The three coordinate axes determine the three **coordinate planes**. The xy-plane contains the x- and y-axes and its equation is $z = 0$, the xz-plane contains the x- and z-axes and its equation is $y = 0$, The yz-plane contains the y- and z-axes and its equation is $x = 0$. These three coordinate planes divide space into eight parts called **octants**. The **first octant** is determined by positive axes.



Take a point P in space, let a be directed distance from yz-plane to P, b be directed distance from xz-plane to P, and c be directed distance from xy-plane to P. We represent the point P by the ordered triple (a, b, c) of real numbers, and we call a, b, and c the **coordinates** of P. The point $P(a, b, c)$ determine a rectangular box. If we drop a perpendicular from P to the xy-plane, we get a point $Q(a, b, 0)$ called the **projection** of P on the xy-plane. Similarly, $R(0, b, c)$ and $S(a, 0, c)$ are the projections of P on the yz-plane and xz-plane, respectively.

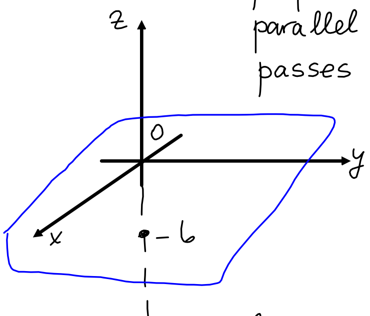


The Cartesian product $\mathbb{R} \times \mathbb{R} \times \mathbb{R} = \mathbb{R}^3 = \{(x, y, z) | x, y, z \in \mathbb{R}\}$ is the set of all ordered triplets of real numbers. We have given a one-to-one correspondence between points P in space and ordered triplets (a, b, c) in $\mathbb{R}^3$. It is called a **tree-dimensional rectangular coordinate system.**

Example 1. What surfaces in $\mathbb{R}^3$ represented by the following equations?

(a) $z = -6$

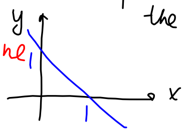
plane
perpendicular to the z-axis
parallel to the (xy)-plane
passes through (0,0,-6)

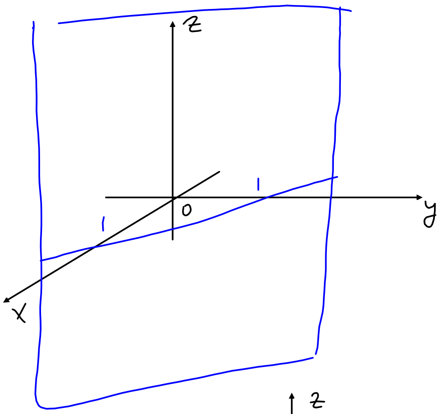


(b) $x + y = 1$

plane
parallel to
the z-axis

(xy)-plane

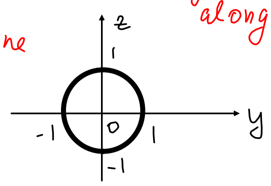


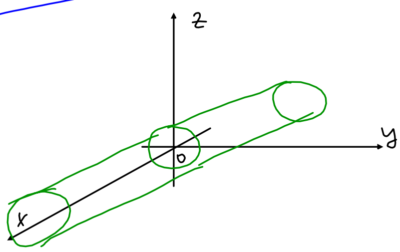


(c) $y^2 + z^2 = 1$

cylinder
along x-axis

yz-plane





The distance formula in three dimensions The distance $|P_1P_2|$ between the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

The **midpoint** of the line segment from $P_1(x_1, y_1, z_1)$ to $P_2(x_2, y_2, z_2)$ is

$$P_M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

Equation of a sphere of radius R and center $C(a, b, c)$ is

$$(x - a)^2 + (y - b)^2 + (z - c)^2 = R^2$$

Example 2. Find an equation of a sphere that has center $C(-1, 2, 4)$ and passes through the point $(-1, 1, -2)$.

$$R = \sqrt{(-1-1)^2 + (1-2)^2 + (-2-4)^2}$$
$$= \sqrt{4+1+36}$$
$$= \sqrt{41}$$

Equation: $\boxed{(x+1)^2 + (y-2)^2 + (z-4)^2 = 41}$