Chapter 12. **Partial derivatives.** Section 12.1 **Functions of several variables.**

Definition. Let $D \subset \mathbb{R}^2$. A function f of two variables is a rule that assigns to each ordered pair (x, y) in D a unique real number denoted by f(x, y). The set D is the **domain** of f and its **range** is the set of values that f takes on, that is, $\{f(x, y) | (x, y) \in D\}$.

We write z = f(x, y) to make explicit the value taken on by f at the general point (x, y). The variables x and y are **independent variables** and z is **dependent variable**.

If a function f is given by a formula and no domain is specified, then the domain of f is understood to be the set of all pairs (x, y) for which the given expression is well-defined real number.

Example 1. Find the domain and the range of the function $f(x,y) = x^2 \ln(x-y)$ and evaluate f(e,0).

Definition. If f is a function of two variables with domain D, the graph of f is the set

$$S = \{(x, y, z) \in \mathbb{R}^3 | z = f(x, y), (x, y) \in D\}.$$

Example 2. Sketch the graph of the function $f(x,y) = 3 - x^2 - y^2$.

Definition. The **level curves** of a function f of two variables are the curves with equations f(x, y) = k, where k is a constant (in the range of f).

A level curve f(x,y) = k is the locus of all points at which f takes on a given value k. In other words, it shows where the graph of f has height k.

Example 3. Describe the level curves for the following functions.

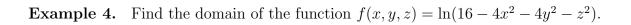
1.
$$f(x,y) = -x + 4y$$

2.
$$f(x,y) = x^2 - y^2$$

Functions of three or more variables.

A function of three variables, f, is a rule that assigns to each ordered triple (x, y, z) in a domain $D \subset \mathbb{R}^3$ a unique real number denoted by f(x, y, z).

We can get some information about f by examining its **level surfaces**, which are surfaces with equations f(x, y, z) = k, where k is a constant. If the point (x, y, z) moves along a level surface, the value of f(x, y, z) remains fixed.



Example 5. Describe the level surfaces of the function $f(x, y, z) = x^2 - y^2 + z^2$.

A function of n variables is a rule that assigns a number $z = f(x_1, x_2, ...x_n)$ to an n-tuple $(x_1, x_2, ...x_n)$ of real numbers. The notation

$$f:D\subset\mathbb{R}^n\to\mathbb{R}$$

is used to signify that f is a real valued function whose domain D is a subset of \mathbb{R}^n .