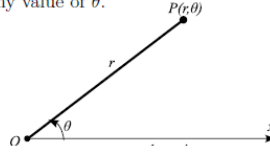


Chapter 13. Multiple integrals.
Section 13.4 Polar coordinates.

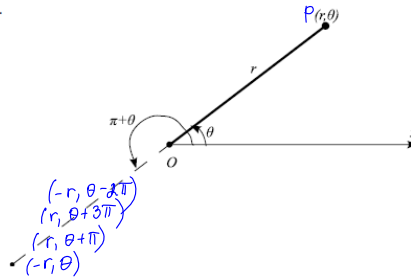
We choose a point in the plane that is called the **pole** (or origin) and labeled O . Then we draw a ray (half-line) starting at O called the **polar axis**. This axis is usually drawn horizontally to the right and corresponds to the positive x -axis in Cartesian coordinates.

If P is any point in the plane, let r be the distance from O to P and let θ be the angle (in radians) between the polar axis and the line OP . Then the point P is represented by the ordered pair (r, θ) and r, θ are called **polar coordinates** of P .

We use the convention that an angle is positive if measured in the counterclockwise direction from the polar axis and negative in the clockwise direction. If $P = O$, then $r = 0$ and we agree that $(0, \theta)$ represents the pole for any value of θ .



We extend the meaning of polar coordinates (r, θ) to the case in which r is negative by agreeing that the points $(-r, \theta)$ and (r, θ) lie on the same line through O and at the same distance $|r|$ from O , but on opposite sides of O .



In the Cartesian coordinate system every point has only one representation, but in the polar coordinate system each point has many representations. Since a complete counterclockwise rotation is given by an angle 2π , the point represented by polar coordinates (r, θ) is also represented by

$$(r, \theta + 2\pi n) \quad \text{and} \quad (-r, \theta + (2n + 1)\pi),$$

where n is any integer.

The connection between polar and Cartesian coordinates is

$$x = r \cos \theta \quad y = r \sin \theta$$

and

$$r^2 = x^2 + y^2 \quad \tan \theta = \frac{y}{x}$$

Equation for θ does not uniquely determine it when x and y are given. Therefore, in converting from Cartesian to polar coordinates, it is not good enough just to find r and θ that satisfy equations. We must choose θ so that the point (r, θ) lies in correct quadrant.

1

The **graph of a polar equation** $r = f(\theta)$, or more generally, $F(r, \theta) = 0$, consists of all points P that have at least one polar representation (r, θ) whose coordinates satisfy the equation.

Note that:

1. If a polar equation is unchanged when θ is replaced by $-\theta$, the curve is symmetric about the polar axis.
2. If the equation is unchanged when r is replaced by $-r$, the curve is symmetric about the pole.
3. If the equation is unchanged when θ is replaced by $\pi - \theta$, the curve is symmetric about the vertical line $\theta = \pi/2$.

Example 1. Sketch the curve of each polar equation

1. $r(\theta) = (\sin \theta) r$

$r^2 = r \sin \theta$

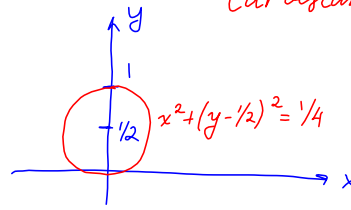
$x^2 + y^2 = y$

$x^2 + (y^2 - y) = 0$

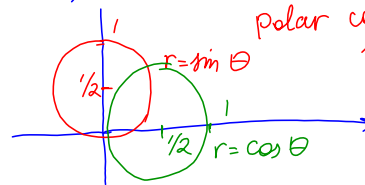
Complete squares

$x^2 + (y - 1/2)^2 = 1/4$ circle centered at $(0, 1/2)$ of radius $1/2$

$x = r \cos \theta$
 $y = r \sin \theta$
 $r^2 = x^2 + y^2$



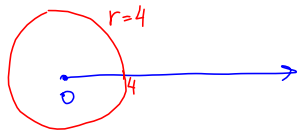
Cartesian coordinate system.



polar coordinate system.

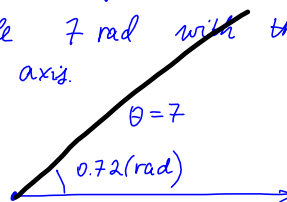
$r = \cos \theta$ circle of radius $1/2$ centered at $(1/2, 0)$

- $r = 4$ circle of radius 4 centered at the pole.

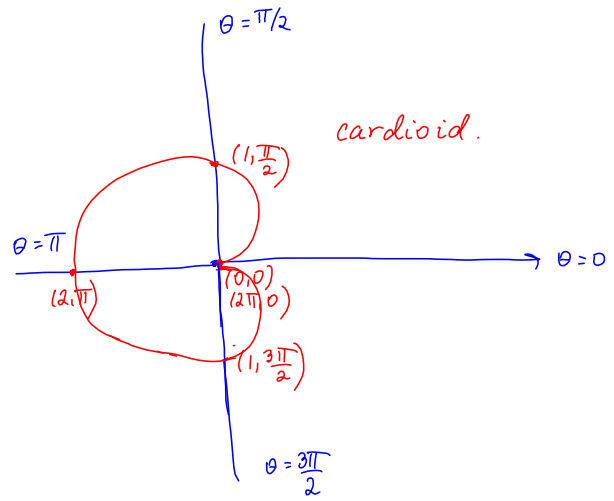
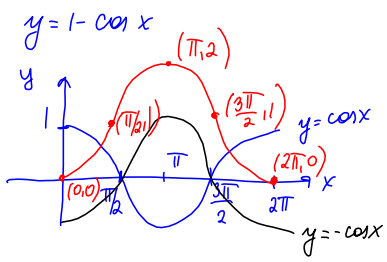


- $\theta = 7$ line that passes through the pole and make the angle 7 rad with the positive polar axis.

$\theta = 2\pi + 0.72$



2. $r = 1 - \cos \theta$



3. $r = \sin 2\theta$