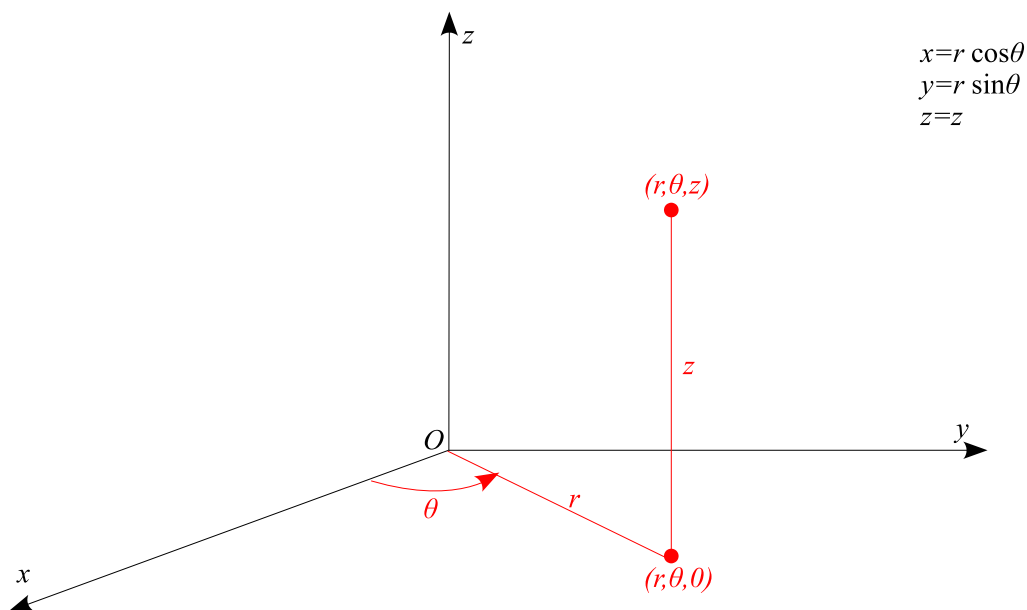


Chapter 13. Multiple integrals.  
Section 13.9 Cylindrical and spherical coordinates.

Cylindrical coordinate system:



To convert from rectangular to cylindrical coordinates we use

$$r^2 = x^2 + y^2 \quad \tan \theta = \frac{y}{x} \quad z = z$$

**Example 1.**

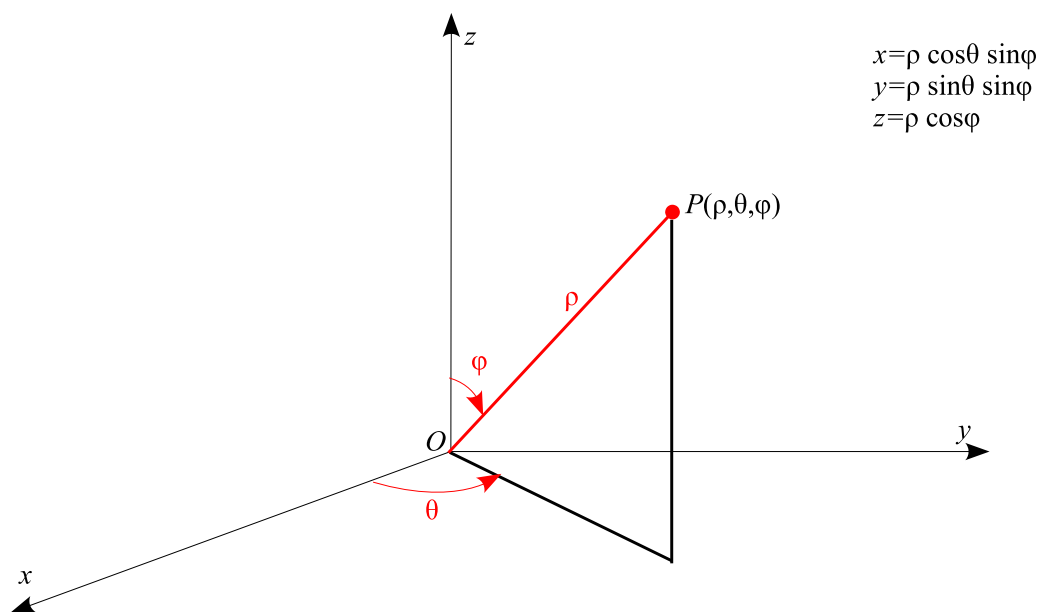
(a) Plot the point with cylindrical coordinates  $(2, 2\pi/3, 8)$  and find its rectangular coordinates.

(b) Find the cylindrical coordinates of the point with rectangular coordinates  $(-\sqrt{2}, \sqrt{2}, 0)$ .

**Example 2.** Sketch the solid given by the inequalities

$$0 \leq \theta \leq \pi/2, \quad r \leq z \leq 2$$

**Spherical coordinate system:**



To convert from rectangular to cylindrical coordinates we use

$$\rho^2 = x^2 + y^2 + z^2 \quad \cos \varphi = \frac{z}{\rho} \quad \cos \theta = \frac{x}{\rho \sin \varphi}$$

**Example 3.**

1. The point  $(1, \pi/4, \pi/6)$  is given in spherical coordinates. Find its rectangular coordinates.

2. The point  $(-\sqrt{3}, -3, -2)$  is given in rectangular coordinates. Find its spherical coordinates.

To convert from spherical to cylindrical coordinates we use

$$\theta = \theta \quad z = \rho \cos \varphi \quad r = \sqrt{\rho^2 - z^2}$$

To convert from cylindrical to spherical coordinates we use

$$\theta = \theta \quad \rho = \sqrt{r^2 + z^2} \quad \cos \varphi = \frac{z}{\rho}$$

**Example 4.**

1. The point  $(8, \pi/6, \pi/2)$  is given in spherical coordinates. Find its cylindrical coordinates.

2. The point  $(\sqrt{2}, \pi/4, 0)$  is given in cylindrical coordinates. Find its spherical coordinates.

**Example 5.** Sketch the solid described by the inequalities

$$-\pi/2 \leq \theta \leq \pi/2, \quad 0 \leq \varphi \leq \pi/6, \quad 0 \leq \rho \leq \sec \varphi$$

**Example 6.** Write the equation

$$x^2 - y^2 - 2z^2 = 4$$

in cylindrical and spherical coordinates.