Section 14.1 Vector fields

Definition. Let D be a set in $\mathbb{R}^2$ (a plane region). A vector field on $\mathbb{R}^2$ is a function $\vec{F}$ that assigns to each point $(x, y) \in D$ a two-dimensional vector $\vec{F}(x, y)$.

$$\vec{F}(x,y) = P(x,y)\vec{\imath} + Q(x,y)\vec{\jmath}$$

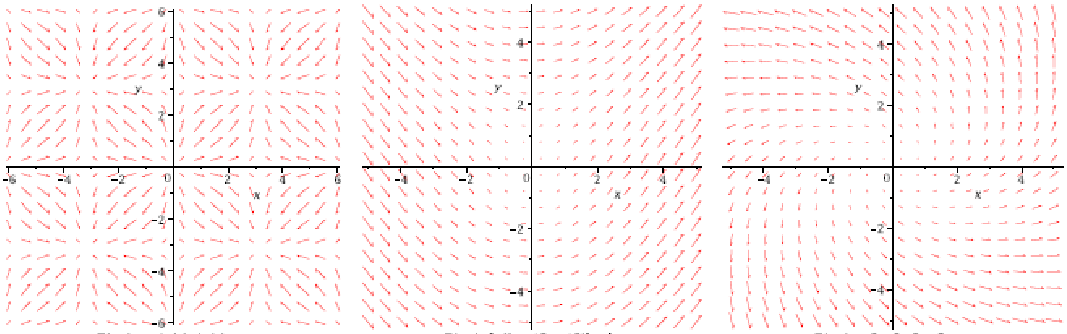
The **components functions** P and Q are sometimes called **scalar fields**.

Definition. Let E be a set in $\mathbb{R}^3$. A vector field on $\mathbb{R}^3$ is a function $\vec{F}$ that assigns to each point $(x, y, z) \in E$ a two-dimensional vector $\vec{F}(x, y, z)$.

$$\vec{F}(x,y,z) = P(x,y,z)\vec{\imath} + Q(x,y,z)\vec{\jmath} + R(x,y,z)\vec{k}$$

$\vec{F}$ is continuous is and only if P, Q, and R are continuous.

Examples of vector fields.



$F(x,y)=\langle \sin(x),\sin(y)\rangle$ $\quad$ $F(x,y)=\ln(1+x^2+y^2)i+xj$ $\quad$ $F(x,y)=\langle 2x-3y,2x+3y\rangle$

Example 1. Sketch the vector field $\vec{F}$ if $\vec{F}(x,y) = x\vec{\imath} - y\vec{\jmath}$.

$= \langle x, -y \rangle$

$F(0,0) = \langle 0,0 \rangle$

$F(0,1) = \langle 0,-1 \rangle$

$F(1,0) = \langle 1,0 \rangle$

$F(0,-1) = \langle 0,1 \rangle$

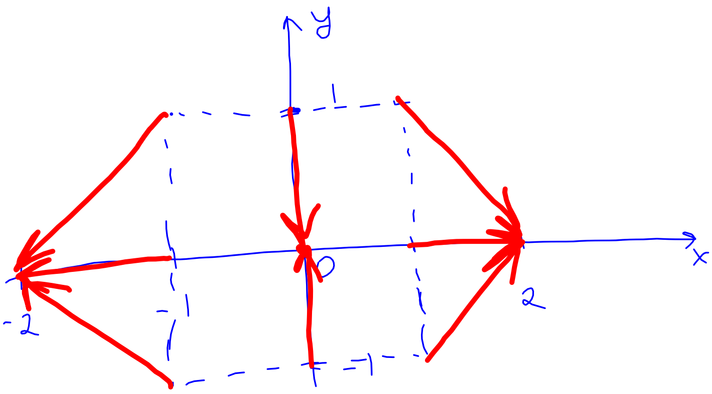
$F(-1,0) = \langle -1,0 \rangle$

$F(1,1) = \langle 1,-1 \rangle$

$F(1,-1) = \langle 1,1 \rangle$

$F(-1,1) = \langle -1,-1 \rangle$

$F(-1,-1) = \langle -1,1 \rangle$



Let $f(x, y)$ be a scalar function of two variables, then

$$\nabla f(x, y) =< f_x, f_y >$$

is a vector field called a **gradient vector field**.

If $f(x, y, z)$ be a scalar function of three variables, then its gradient vector field is defined as

$$\nabla f(x, y, z) =< f_x, f_y, f_z >$$

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Example 2. Find the gradient vector field of the function $f(x, y, z) = x \ln(y - z)$.

$$\nabla f = < f_x, f_y, f_z >$$

$$= \boxed{< \ln(y-z), \frac{x}{y-z}, -\frac{x}{y-z} >}$$

A vector field is called a conservative vector field if it is the gradient of some scalar function, that it. if there exists a function f such that $\vec{F} = \nabla f$. Then f is called a potential function for $\vec{F}$.