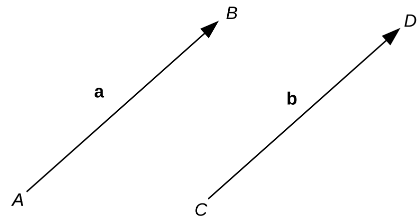
Section 12.2 **Vectors.**

The term **vector** indicates a quantity that has both direction and magnitude. A vector is represented by an arrow or a directed line segment.



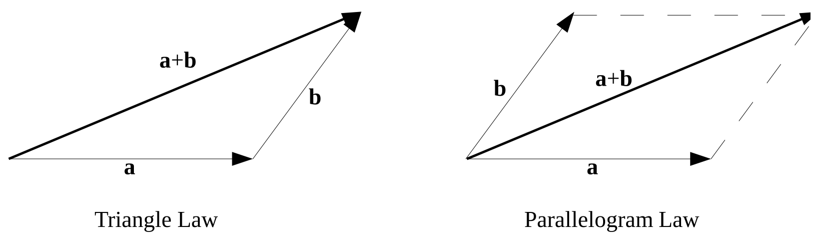
If a particle moves from a point A to a point B, then the corresponding displacement vector **a** has **initial point** A and **terminal point** B and we write $\mathbf{a} = \overrightarrow{AB}$. The directed line segment $\overrightarrow{AB}$ is a **representation** of the vector **a**.

Two vectors that have the same length and direction are called **equivalent** or **equal** even if they have different initial and terminal points. Since $|AB| = |CD|$ adb both segments have the same direction, $\mathbf{a} = \mathbf{b}$.

The only vector with length 0 is the **zero vector 0**. This vector is the only vector with no specific direction.

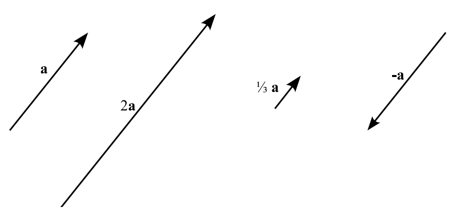
Vector addition.

If two vectors **a** and **b** positioned so the initial point of **b** is at the terminal point of **a**, then the **sum** $\mathbf{a} + \mathbf{b}$ is the vector from the initial point of **a** to the terminal point of **b**.



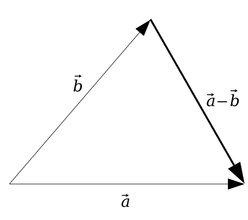
Scalar multiplication.

If c is a scalar and **a** is a vector, then $c\mathbf{a}$ is a vector whose length is $|c|$ times the length of **a** and has the same direction as **a** ($c > 0$) or the opposite direction to **a** ($c < 0$). If either $c = 0$ or $\mathbf{a} = \mathbf{0}$, then $c\mathbf{a} = \mathbf{0}$.



Two vectors **a** and **b** are called **parallel** if $\mathbf{b} = c\mathbf{a}$ for some scalar c.

Vector difference. $\mathbf{a} - \mathbf{b} = \mathbf{a} + (-1)\mathbf{b}$



The **magnitude (length)** $|\mathbf{a}|$ of $\mathbf{a}$ is the length of any its representation.

A **unit vector** is a vector whose length is 1.

A vector $\mathbf{u} = \frac{1}{|\mathbf{a}|}\mathbf{a}$ is a unit vector in the direction of $\mathbf{a}$.

Properties of vectors

If $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are vectors and k and m are scalars, then

1. $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$
2. $\mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$
3. $\mathbf{a} + \mathbf{0} = \mathbf{a}$
4. $\mathbf{a} + (-\mathbf{a}) = \mathbf{0}$
5. $k(\mathbf{a} + \mathbf{b}) = k\mathbf{a} + k\mathbf{b}$
6. $(k + m)\mathbf{a} = k\mathbf{a} + m\mathbf{a}$
7. $(km)\mathbf{a} = k(m\mathbf{a})$
8. $1\mathbf{a} = \mathbf{a}$

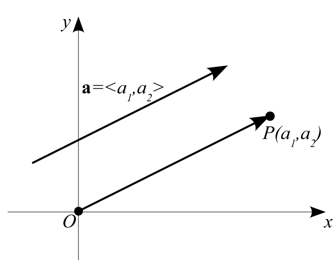
Components.

2D

A **two-dimensional vector** is an ordered pair $\mathbf{a} =< a_1, a_2 >$ of real numbers. The numbers a_1 and a_2 are called the **components** of $\mathbf{a}$.

A **representation** of the vector $\mathbf{a} =< a_1, a_2 >$ is a directed line segment $\overrightarrow{AB}$ from any point $A(x, y)$ to the point $B(x + a_1, y + a_2)$.

A particular representation of $\mathbf{a} =< a_1, a_2 >$ is the directed line segment $\overrightarrow{OP}$ from the origin to the point $P(a_1, a_2)$, and $\mathbf{a} =< a_1, a_2 >$ is called the **position vector** of the point $P(a_1, a_2)$.



If $A(x_1, y_1)$ and $B(x_2, y_2)$, then

$$\overrightarrow{AB} =< x_2 - x_1, y_2 - y_1 >$$

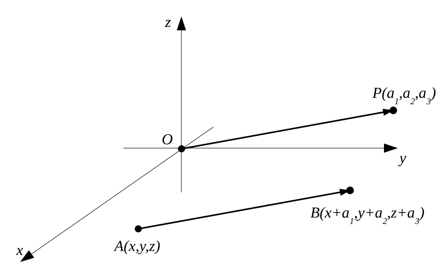
$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2}$$

3D

A **tree-dimensional vector** is an ordered triple $\mathbf{a} =< a_1, a_2, a_3 >$ of real numbers. The numbers a_1, a_2, and a_3 are called the **components** of $\mathbf{a}$.

A **representation** of the vector $\mathbf{a} =< a_1, a_2, a_3 >$ is a directed line segment $\overrightarrow{AB}$ from any point $A(x, y, z)$ to the point $B(x + a_1, y + a_2, z + a_3)$.

A particular representation of $\mathbf{a} =< a_1, a_2, a_3 >$ is the directed line segment $\overrightarrow{OP}$ from the origin to the point $P(a_1, a_2, a_3)$, and $\mathbf{a} =< a_1, a_2, a_3 >$ is called the **position vector** of the point $P(a_1, a_2, a_3)$.



If $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$, then

$$\overrightarrow{AB} =< x_2 - x_1, y_2 - y_1, z_2 - z_1 >$$

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

2D

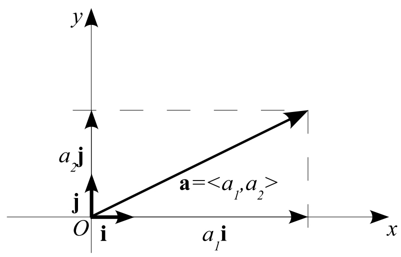
$\mathbf{a}+\mathbf{b} =< a_1+b_1, a_2+b_2 >$

$c\mathbf{a} =< ca_1, ca_2 >$

$\mathbf{a}-\mathbf{b} =< a_1-b_1, a_2-b_2 >$

$\mathbf{i} =< 1, 0 >$, $\mathbf{j} =< 0, 1 >$.

$\mathbf{a} =< a_1, a_2 >= a_1\mathbf{i}+a_2\mathbf{j}$



3D

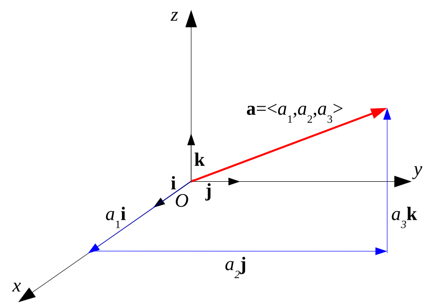
$\mathbf{a}+\mathbf{b} =< a_1+b_1, a_2+b_2, a_3+b_3 >$

$c\mathbf{a} =< ca_1, ca_2, ca_3 >$

$\mathbf{a}-\mathbf{b} =< a_1-b_1, a_2-b_2, a_3-b_3 >$

$\mathbf{i} =< 1, 0, 0 >$, $\mathbf{j} =< 0, 1, 0 >$, $\mathbf{k} =< 0, 0, 1 >$

$\mathbf{a} =< a_1, a_2, a_3 >= a_1\mathbf{i}+a_2\mathbf{i}+a_3\mathbf{k}$



Example 1. Find $|2\mathbf{a}-5\mathbf{b}|$ if

1. $\mathbf{a} =< -1, 2 >$, $\mathbf{b} =< -2, -1 >$

2. $\mathbf{a} =< 3, -1, 2 >$, $\mathbf{b} =< 4, 2, -1 >$

Example 2. Find the unit vector in the direction of the given vector.

1. $\mathbf{a} = 5\mathbf{i} - 3\mathbf{j}$

2. $\mathbf{b} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$.