

Section 13.4. Motion in space: velocity and acceleration

Definition. If $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$ is a vector function representing the position of a particle at time t , then

velocity at time t is

$$\mathbf{v}(t) = \mathbf{r}'(t) = \langle x'(t), y'(t), z'(t) \rangle$$

speed at time t is

$$s = |\mathbf{v}(t)| = \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}$$

acceleration at time t is

$$\mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{r}''(t) = \langle x''(t), y''(t), z''(t) \rangle$$

Example 1. The vector function $\vec{r}(t) = \langle t^2 + t, t^2 - t, t^3 \rangle$ represents the position of a particle at time t . Find the velocity, acceleration and the speed.

Example 2. Find the velocity and position vectors of a particle that has the acceleration $\mathbf{a}(t) = \sin t \mathbf{i} + 2 \cos t \mathbf{j} + 6t \mathbf{k}$ with the initial velocity $\mathbf{v}(0) = -\mathbf{k}$ and initial position $\mathbf{r}(0) = \mathbf{j} - 4\mathbf{k}$.