Section 13.4. **Motion in space: velocity and acceleration**

Definition. If $\mathbf{r}(t) =< x(t), y(t), z(t) >$ is a vector function representing the position of a particle at time t, then
velocity at time t is

$$\mathbf{v}(t) = \mathbf{r}'(t) =< x'(t), y'(t), z'(t) >$$

speed at time t is

$$s = |\mathbf{v}(t)| = \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}$$

acceleration at time t is

$$\mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{r}''(t) =< x''(t), y''(t), z''(t) >$$

Example 1. The vector function $\vec{r}(t) =< t^2 + t, t^2 - t, t^3 >$ represents the position of a particle at time t. Find the velocity, acceleration and the speed.

velocity $\vec{v}(t) = \vec{r}\,'(t) = <2t+1, 2t-1, 3t^2>$

speed $s(t) = |\vec{v}(t)| = \sqrt{(2t+1)^2 + (2t-1)^2 + 9t^4} = \sqrt{8t^2 + 9t^4 + 2}$

acceleration $\vec{a}(t) = \vec{v}\,'(t) = <2, 2, 6t>$

Example 2. Find the velocity and position vectors of a particle that has the acceleration $\mathbf{a}(t) = \sin t\mathbf{i} + 2\cos t\mathbf{j} + 6t\mathbf{k}$ with the initial velocity $\mathbf{v}(0) = -\mathbf{k}$ and initial position $\mathbf{r}(0) = \mathbf{j} - 4\mathbf{k}$.

$\vec{a}(t) = <\sin t, 2\cos t, 6t>$, $\vec{v}(0) = <0, 0, -1>$, $\vec{r}(0) = <0, 1, -4>$

$\vec{v}(t) = \int \vec{a}(t)\,dt = <\int \sin t\,dt, 2\int \cos t\,dt, \int 6t\,dt>$

$\vec{v}(t) = <-\cos t + C_1, 2\sin t + C_2, 3t^2 + C_3>$

Plug in $t=0$: $\vec{v}(0) = <-1 + C_1, C_2, C_3> = <0, 0, -1>$

$-1 + C_1 = 0 \Rightarrow \boxed{C_1 = 1}$

$\boxed{C_2 = 0}$

$\boxed{C_3 = -1}$

$\boxed{\vec{v}(t) = <-\cos t + 1, 2\sin t, 3t^2 - 1>}$

$\vec{r}(t) = \int \vec{v}(t)\,dt = <\int(-\cos t + 1)\,dt, 2\int \sin t\,dt, \int(3t^2 - 1)\,dt>$

$= <-\sin t + t + K_1, -2\cos t + K_2, t^3 - t + K_3>$

K_1, K_2, K_3 are constants.

Plug in $t=0$: $\vec{r}(0) = <K_1, -2 + K_2, K_3> = <0, 1, -4>$

$\boxed{K_1 = 0}$ $-2 + K_2 = 1$, $\boxed{K_3 = -4}$

$\boxed{K_2 = 3}$

$\boxed{\vec{r}(t) = <-\sin t + t, -2\cos t + 3, t^3 - t - 4>}$