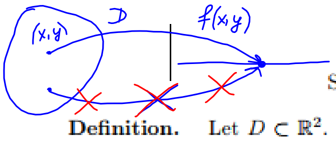


Chapter 14. **Partial derivatives.**
Section 14.1 **Functions of several variables.**

Definition. Let $D \subset \mathbb{R}^2$. A function f of two variables is a rule that assigns to each ordered pair (x, y) in D a unique real number denoted by $f(x, y)$. The set D is the domain of f and its range is the set of values that f takes on, that is, $\{f(x, y) | (x, y) \in D)\}$.

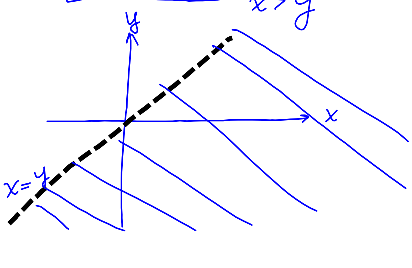
We write $z = f(x, y)$ to make explicit the value taken on by f at the general point (x, y). The variables x and y are independent variables and z is dependent variable.

If a function f is given by a formula and no domain is specified, then the domain of f is understood to be the set of all pairs (x, y) for which the given expression is well-defined real number.

Example 1. Find the domain and the range of the function $f(x, y) = x^2 \ln(x - y)$ and evaluate $f(e, 0)$.

$\boxed{\text{Domain: } x-y>0}$

$x>y$



$\boxed{\text{Range: } (-\infty, \infty)}$

$f(x,y)= x^2 \ln(x-y)$

$f(e,0) = e^2 \ln(e-0)$

$= e^2 \cancel{\ln e}$

$= \boxed{e^2}$

Definition. If f is a function of two variables with domain D, the **graph** of f is the set

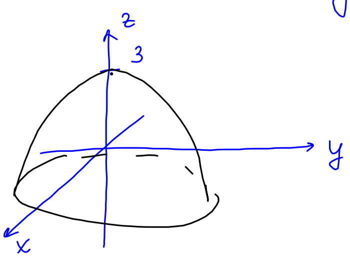
$$S = \{(x, y, z) \in \mathbb{R}^3 | z = f(x, y), (x, y) \in D\}.$$

Example 2. Sketch the graph of the function $f(x, y) = 3 - x^2 - y^2$.

$z = 3 - x^2 - y^2$ – circular paraboloid

vertex @ $(0,0,3)$

opens down



level curves are traces of the surface $z=f(x,y)$ in $z=k$

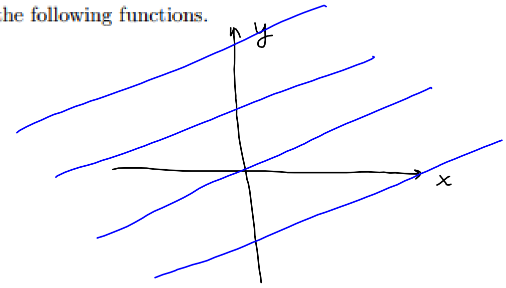
Definition. The **level curves** of a function f of two variables are the curves with equations $f(x,y)=k$, where k is a constant (in the range of f).

A level curve $f(x,y)=k$ is the locus of all points at which f takes on a given value k. In other words, it shows where the graph of f has height k.

Example 3. Describe the level curves for the following functions.

1. $f(x,y)=-x+4y$

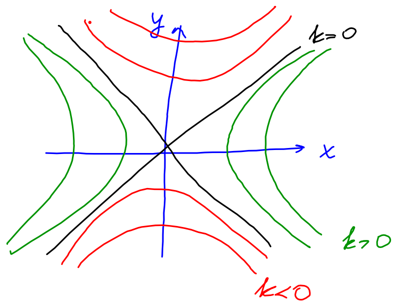
$k=-x+4y$ – lines.
k is a constant



2. $f(x,y)=x^2-y^2$

$k=x^2-y^2$, $k\neq 0$ hyperbolas

$k=0=x^2-y^2 \Rightarrow \left.\begin{array}{l} x=y \\ x=-y \end{array}\right\}$ lines



Functions of three or more variables.

A function of three variables, f, is a rule that assigns to each ordered triple (x,y,z) in a domain $D\subset\mathbb{R}^3$ a unique real number denoted by $f(x,y,z)$.

We can get some information about f by examining its **level surfaces**, which are surfaces with equations $f(x,y,z)=k$, where k is a constant. If the point (x,y,z) moves along a level surface, the value of $f(x,y,z)$ remains fixed.

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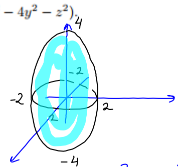
Example 4. Find the domain of the function $f(x,y,z)=\ln(16-4x^2-4y^2-z^2)$.

$16-4x^2-4y^2-z^2>0$

$\frac{4x^2+4y^2+z^2}{16}<\frac{16}{16}$

$\frac{x^2}{4}+\frac{y^2}{4}+\frac{z^2}{16}<1$



the inside of the ellipsoid $\frac{x^2}{4}+\frac{y^2}{4}+\frac{z^2}{16}=1$,
the ellipsoid is not included

Example 5. Describe the level surfaces of the function $f(x,y,z)=x^2-y^2+z^2$.

$k=x^2-y^2+z^2$

$k=0$: $x^2-y^2+z^2=0$ cone

$k>0$: $k=x^2-y^2+z^2$ hyperboloid on one sheet

$k<0$: $k=x^2-y^2+z^2$ hyperboloid of two sheets.

A **function of n variables** is a rule that assigns a number $z = f(x_1, x_2, ...x_n)$ to an n-tuple $(x_1, x_2, ...x_n)$ of real numbers. The notation

$$f : D \subset \mathbb{R}^n \to \mathbb{R}$$

is used to signify that f is a real valued function whose domain D is a subset of $\mathbb{R}^n$.