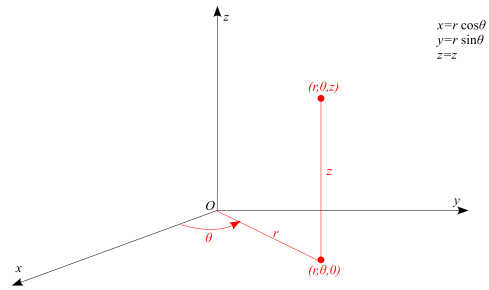
Section 15.7 **Triple integrals in cylindrical coordinates.**

Cylindrical coordinate system:



To convert from rectangular to cylindrical coordinates we use

$$r^2 = x^2 + y^2 \qquad \tan\theta = \frac{y}{x} \qquad z = z$$

Example 1.
(a) Plot the point with cylindrical coordinates $(2, 2\pi/3, 8)$ and find its rectangular coordinates.

(b) Find the cylindrical coordinates of the point with rectangular coordinates $(-\sqrt{2}, \sqrt{2}, 0)$.

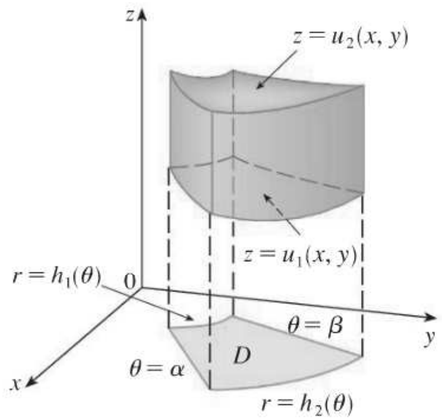
Example 2. Sketch the solid given by the inequalities

$$0 \le \theta \le \pi/2, \qquad r \le z \le 2$$

Suppose that E is a type 1 region whose projection D on the xy-plane is described in polar coordinates

$$E = \{(x,y,z)|(x,y) \in D, \varphi_1(x,y) \le z \le \varphi_1(x,y)\}$$

$$D = \{(r,\theta)|\alpha \le \theta \le \beta, h_1(\theta) \le r \le h_2(\theta)\}$$



Then

$$\iiint\limits_E f(x,y,z)dV = \iint\limits_D \left[\int_{\varphi_1(x,y)}^{\varphi_2(x,y)} f(x,y,z)dz\right] dA$$

If we switch to cylindrical coordinates

$$x = r\cos\theta, \qquad y = r\sin\theta, \qquad z = z, \qquad dA = r\,dr\,d\theta$$

we will get

$$\boxed{\iiint\limits_E f(x,y,z)dV = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} \int_{\varphi_1(r\cos\theta, r\sin\theta)}^{\varphi_2(r\cos\theta, r\sin\theta)} f(r\cos\theta, r\sin\theta, z)\, r\,dz\,dr\,d\theta}$$

Example 3. Sketch the solid whose volume is given by $\displaystyle\int_0^{2\pi}\int_0^2\int_0^{4-r^2} r\,dz\,dr\,d\theta.$

Example 4. Evaluate $\iiint_E y dV$ where E is the solid that lies between the cylinders $x^2+y^2=1$ and $x^2+y^2=4$, above the xy-plane, and below the plane $z=x+2$.