Chapter 16. **Vector calculus.**
Section 16.1 **Vector fields.**

Definition. Let D be a set in $\mathbb{R}^2$ (a plane region). A **vector field on** $\mathbb{R}^2$ is a function $\mathbf{F}$ that assigns to each point $(x, y) \in D$ a two-dimensional vector $\mathbf{F}(x, y)$.

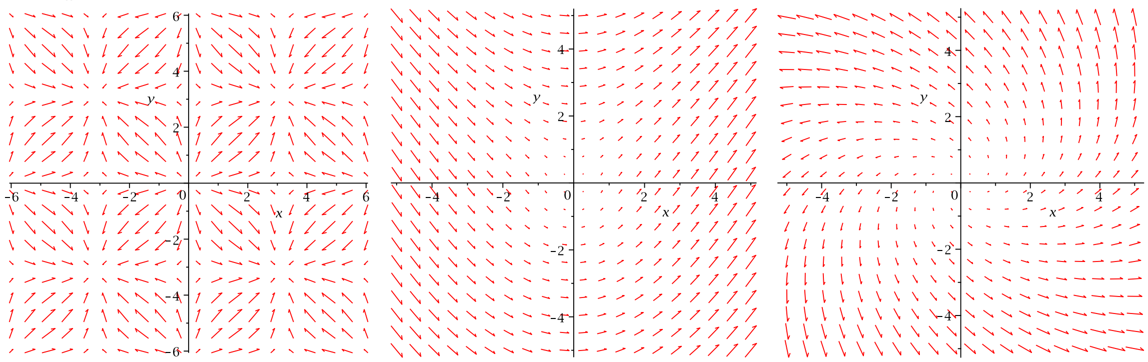
$$\mathbf{F}(x, y) = P(x, y)\mathbf{i} + Q(x, y)\mathbf{j}$$

The **components functions** P and Q are sometimes called **scalar fields**.

Definition. Let E be a set in $\mathbb{R}^3$. A **vector field on** $\mathbb{R}^3$ is a function $\mathbf{F}$ that assigns to each point $(x, y, z) \in E$ a two-dimensional vector $\mathbf{F}(x, y, z)$.

$$\mathbf{F}(x, y, z) = P(x, y, z)\mathbf{i} + Q(x, y, z)\mathbf{j} + R(x, y, z)\mathbf{k}$$

$\mathbf{F}$ is continuous is and only if P, Q, and R are continuous.
Examples of vector fields.



F(x,y)=<sin(x),sin(y)> F(x,y)=ln(1+x^2+y^2)**i**+x**j** F(x,y)=<2x−3y,2x+3y>

Example 1. Sketch the vector field $\mathbf{F}$ if $\mathbf{F}(x, y) = x\mathbf{i} - y\mathbf{j}$.

Let $f(x, y)$ be a scalar function of two variables, then

$$\nabla f(x, y) = < f_x, f_y >$$

is a vector field called a **gradient vector field**.

If $f(x, y, z)$ be a scalar function of three variables, then its gradient vector field is defined as

$$\nabla f(x, y, z) = < f_x, f_y, f_z >$$

Example 2. Find the gradient vector field of the function $f(x, y, z) = x \ln(y - z)$.

A vector field is called a **conservative vector field** if it is the gradient of some scalar function, that it. if there exists a function f such that $\mathbf{F} = \nabla f$. Then f is called a **potential function** for $\mathbf{F}$.