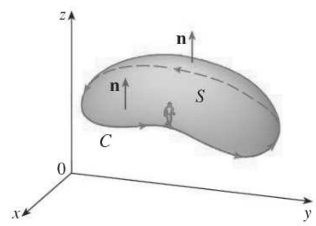
Section 16.8 **Stokes' Theorem.**

Stokes' Theorem. Let S be an oriented piecewise-smooth surface that is bounded by a simple, closed, piecewise-smooth boundary curve C with positive orientation. Let $\mathbf{F}$ be a vector field whose components have continuous partial derivatives on an open region in $\mathbb{R}^3$ that contains S. Then

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$$



The Stokes' Theorem says that the line integral around the boundary curve of S of the tangential component of $\mathbf{F}$ is equal to the surface integral of the normal component of the curl of $\mathbf{F}$.

Example 1. Use Stokes' Theorem to evaluate $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$ if $\mathbf{F}(x, y, z) =< xyz, x, e^{xy}\cos(z) >$ and S is hemisphere $x^2 + y^2 + z^2 = 1$, oriented upward.

Example 2. Use Stokes' Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ if $\mathbf{F}(x, y, z) =< z^2, y^2, xy >$ and C is the triangle with vertices (1,0,0), (0,1,0), and (0,0,2) and is oriented counterclockwise as viewed from above.

Example 3. Verify Stokes' Theorem for the surface S: $x^2 + y^2 + 5z = 1$, $z \geq -5$ (oriented by upward normal) and the vector field $\mathbf{F} = xz\mathbf{i} + yz\mathbf{j} + (x^2 + y^2)\mathbf{k}$.