Section 16.9 **Divergence Theorem.**

**Divergence theorem.** Let $E$ be a simple solid region whose boundary surface $S$ has positive (outward) orientation. Let $\mathbf{F}$ be a vector field whose component functions have continuous partial derivatives on an open region that contains $E$. Then

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iiint_E \text{div } \mathbf{F} \, dV$$

**Example 1.** Use the Divergence Theorem to calculate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ if $\mathbf{F} =< ye^{z^2}, y^2, e^{xy} >$ and $S$ is the surface of the solid bounded by the cylinder $x^2 + y^2 = 9$ and the planes $z = 0$ and $z = y - 3$.

**Example 2.** Verify the Divergence Theorem for the region

$$E = \{(x, y, z) : 0 \leq z \leq 9 - x^2 - y^2\}$$

and the vector field $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$