Math 251. WEEK in REVIEW 8. Fall 2013

1. Find the gradient vector field of $f(x, y, z) = x \ln(y^4 + z^2)$

2. Let C be the line segment starting at $(0, 1, 1)$ and ending at $(3, 1, 4)$. Find the mass of a thin wire in the shape of C with the density $\rho(x, y) = x + y$.

3. Find the mass of a thin wire in the shape of C with the density $\rho(x, y, z) = 7y^2z$ if C is given by $\vec{r}(t) = \left\langle \frac{2}{3}t^3, t, t^2 \right\rangle$, $0 \leq t \leq 1$.

4. Find the line integral of the vector field $\vec{F}(x, y, z) = \langle -yz^2, xz^2, z^3 \rangle$ around the circle $\vec{r}(t) =< 2\cos t, 2\sin t, 8 >$.

5. Find the work done by the force field $\vec{F}(x, y, z) =< z, x, y >$ in moving a particle from the point $(3, 0, 0)$ to the point $(0, \pi/2, 3)$

   (a) along the straight line

   (b) along the helix $x = 3\cos t,\ y = t,\ z = 3\sin t$

6. Find a scalar function $f(x, y, z)$ such that $\nabla f =< 2xy+z, x^2-2y, x >$ and $f(1, 2, 0) = 3$.

7. Let $\vec{F}(x,y) = \langle x^3y^4, x^4y^3 \rangle$. Compute $\displaystyle\int_C \vec{F} \cdot d\vec{r}$ where

(a) C is any simple closed path.

(b) C is any path from the point $M(0,0)$ to the point $N(1,2)$.

8. Let $\vec{F}(x,y) =< x + y^2, 2xy + y^2 >$.

   (a) Show that $\vec{F}$ is conservative vector field.

   (b) Compute $\displaystyle\int_C \vec{F} \cdot d\vec{r}$ where C is any path from (-1,0) to (2,2).

9. Let $\vec{F}(x,y) =< 2x + y^2 + 3x^2y, 2xy + x^3 + 3y^2 >$.

(a) Show that $\vec{F}$ is conservative vector field.

(b) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where C is the arc of the curve $y = x \sin x$ from $(0,0)$ to $(\pi, 0)$.

10. Compute the integral $I = \oint_C (\cos x^4 + xy)\, dx + (y^2 e^y + x^2)\, dy$, where C is the triangular curve consisting of the line segments from $(0,0)$ to $(1,0)$, from $(1,0)$ to $(1,3)$, and from $(1,3)$ to $(0,0)$.

11. Compute the integral along the given positively oriented curve C:

$$\int_C (y^2 - \arctan x)\,dx + (3x + \sin y)\,dy,$$

where C is the boundary of the region enclosed by the parabola $y = x^2$ and the line $y = 4$.

12. Compute the integral

$$\int_C (12 - x^2 y - y^3 + \tan x)\, dx + (xy^2 + x^3 - e^y)\, dy$$

where C is positively oriented boundary of the region enclosed by the circle $x^2 + y^2 = 4$. Sketch the curve C indicating the positive direction.

13. Given the line integral $I = \int_C 4x^2y\,dx - (2+x)\,dy$ where C consists of the line segment from $(0,0)$ to $(2,-2)$, the line segment from $(2,-2)$ to $(2,4)$, and the part of the parabola $y = x^2$ from $(2,4)$ to $(0,0)$. Use Green's theorem to **evaluate** the given integral and **sketch** the curve C indicating the *positive direction.*