Math 251.   WEEK in REVIEW 1.   Fall 2013

1. a) Find the center and radius of the sphere

$$x^2 + y^2 + z^2 + 4x + 6y - 10z + 2 = 0$$

   b) Find an equation of the sphere given that it touches the $yz-$ plane and has the center at $(2, 1, 3)$.

2. Describe in words the region of $\mathbb{R}^3$ represented by the equation or inequality.

   (a) $y = z$

   (b) $y > 2$

   (c) $y^2 + z^2 \le 4$

   (d) $x^2 + y^2 + z^2 - 2z < 3$

3. Find the area of the triangle with the vertices $P(1, 1, 0)$, $Q(1, 0, 1)$, and $R(0, 1, 1)$.

4. a) A constant force of $\vec{F} = 3\vec{\imath} + 2\vec{\jmath} - \vec{k}$ moves an object along the line segment from $(1, 0, 2)$ to $(3, 4, 5)$. Find the work done.

   b) Find the angle between the force and the displacement vectors in question a).

5. Find two unit vectors orthogonal to both $< 1, 0, 1 >$ and $< 2, 3, 4 >$.

6. What restrictions must be made on b so that the vector $2\imath + b\jmath$ is orthogonal to vector $-3\vec{\imath} + 2\vec{\jmath} + \vec{k}$? to vector $\vec{k}$?

7. Find an equation for the plane containing two lines
   $\vec{r} = < 0, 1, -2 > + t < 2, 3, -1 >$ and
   $\vec{r} = < 2, -1, 0 > + t < 2, 3, -1 >$.

8. Find symmetric and parametric equations of the line through the point $M(2, 0, -3)$ and
   a) parallel to the x-axis;

   b) parallel to the line of intersection of the planes $3x - y + 2z - 7 = 0$ and $x + 3y - 2z - 3 = 0$.

9. a) Verify that the given planes
   $2x + y - 3z + 4 = 0$ and $4x + 2y - 6z = 3$
   are parallel and find the distance between them.
   b) Find the distance from the point $(1, 0, -1)$ to the line
   $x = 5 - t$, $y = 3t$, $z = -1 + 2t$.
   c) Verify that the given lines
   $\{3x - y + 2z - 7 = 0, x + 3y - 2z - 3 = 0\}$ and
   $\dfrac{x + 7}{2} = \dfrac{y - 5}{-4} = \dfrac{z - 9}{-5}$
   are parallel and find the distance between them.

10. Find an angle between the planes

$2x + 2y - z = 4$ and $6x - 3y + 2z = 5$.

11. Determine whether the line through $(2, 1, 0)$ and $(1, 0, 1)$ and the line $\dfrac{x}{2} = \dfrac{y - 3}{-1} = \dfrac{z + 5}{2}$ are parallel, skew, or intersecting. If they are intersecting, find the point of intersection and the angle between the lines. If they are parallel or skew, find the distance between the lines.