Math 251. WEEK in REVIEW 11. Fall 2013

1. Use Stokes' Theorem to evaluate $\oint_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x,y,z) = < 3z, 5x, -2y >$ and C is the ellipse in which the plane $z = y + 3$ intersects the cylinder $x^2 + y^2 = 4$, with positive orientation as viewed from above.

2. Find the work performed by the forced field $\vec{F} = \langle -3y^2, 4z, 6x \rangle$ on a particle that traverses the triangle C in the plane $z = \frac{1}{2}y$ with vertices $A(2,0,0)$, $B(0,2,1)$, and $O(0,0,0)$ with a counterclockwise orientation looking down the positive z-axis.

3. Evaluate $I = \oint_C \vec{F} \cdot d\vec{r}$ if $\vec{F} = \langle 2y + 3e^x, z - y^8, x + \ln(z^2+1) \rangle$ and C is the curve of intersection of the plane $x + y + z = 0$ and sphere $x^2 + y^2 + z^2 = 1$. (Orient C to be counterclockwise when viewed from above).

4. Verify Stokes' Theorem for the surface S: $x^2 + y^2 + 5z = 1$, $z \geq -5$ (oriented by upward normal) and the vector field $\vec{F} = xz\vec{\imath} + yz\vec{\jmath} + (x^2+y^2)\vec{k}$.

5. Use the Divergence Theorem to compute $\iint_S \vec{F} \cdot d\vec{S}$, where $\vec{F} = x^3\vec{\imath} + y^3\vec{\jmath} + z^3\vec{k}$ and S is the surface of the region enclosed by $x^2 + y^2 = 1$ and the planes $z = 0$, $z = 2$.

6. Use the Divergence Theorem to find flux of the vector field $\vec{F} = \langle x, y, 1 \rangle$ across the surface S which is the boundary of the region enclosed by the cylinder $y^2 + z^2 = 1$ and the planes $x = 0$ and $x + y = 5$.

7. Verify the Divergence Theorem for the region

$$E = \{(x,y,z) : \ 0 \leq z \leq 9 - x^2 - y^2\}$$

and the vector field $\vec{F} = \vec{r} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$

8. Apply the Divergence Theorem to compute $\iint_S \vec{F} \cdot d\vec{S}$ for the vector field

$$\vec{F}(x,y,z) = \left\langle x^3 + \sin(yz), y^3, y + z^3 \right\rangle$$

over the complete boundary S of the solid hemisphere $\{(x,y,z) : x^2 + y^2 + z^2 \leq 1, z \geq 0\}$ with outward normal.