Math 251. WEEK in REVIEW 3. Fall 2013

1. Let $f(x,y) = \sin(xy) + \pi$. Find $f(x-y, x+y)$.

2. Let $f(x,y,z) = y + xz$. Find $f(y-z, x^3, y+z)$.

3. Find the domain and range of the functions:

   (a) $f(x,y) = \ln(2 - x^2 - y^2)$;

   (b) $f(x,y,z) = e^{-\frac{1}{x^2+y^2+z^2}}$

4. Sketch the graphs of the functions:

(a) $f(x, y) = x^2 + y^2 - 3$

(b) $f(x, y) = \sqrt{4 - x^2 - y^2}$

(c) $f(x, y) = -2x - 4y + 4$

5. Draw several level curves of the functions

(a) $f(x,y) = \sqrt{x-y}$

(b) $f(x,y) = e^{-x^2-y^2}$

6. Describe and sketch several level surfaces of the function $f(x,y,z) = -x^2 - y^2 - z^2$

7. Find the first partial derivatives of the functions

(a) $f(x, y) = y^{x^y}$

(b) $g(x, y, z) = x \cos \dfrac{y}{x+z}$

(c) $h(x, y, z) = xz \tan \left(\dfrac{y}{x}\right)$

8. Find $\dfrac{\partial z}{\partial x}$, $\dfrac{\partial z}{\partial y}$ if

$$xyz = \cos(x + y + z)$$

9. Determine whether the function

$$f(x,y) = e^{-x}\cos y - e^{-y}\cos x$$

satisfies the Laplace equation $f_{xx} + f_{yy} = 0$.

10. Determine whether the function

$$f(t,x) = \cos(x - at) + (x + at)^2$$

satisfies the wave equation $f_{tt} = a^2 f_{xx}$,

11. If $u = x^y$, show that $\dfrac{x}{y}\dfrac{\partial u}{\partial x} + \dfrac{1}{\ln x}\dfrac{\partial u}{\partial y} = 2u.$

12. Where does the plane tangent to the surface $z = e^{x-y}$ at $(1, 1, 1)$ meet the z-axis?

13. Show that the surfaces given by $f(x, y) = x^2 + y^2$ and $g(x, y) = -x^2 - y^2 + xy^3$ have the same tangent plane at $(0, 0)$.

14. Find the differential of the function

$$f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$$

15. Use differentials to estimate

$$\sqrt{(4.01)^2 + (3.98)^2 + (2.02)^2}$$

16. The two legs of a right triangle are measured as 5 m and 12 m respectively, with a possible error in measurement of at most 0.2 cm in each. Use differentials to estimate the maximum error in the calculated value of the area of the triangle.