Math 251. WEEK in REVIEW 4. Fall 2013

1. If $z = y + f(x^2 - y^2)$, where f is differentiable, show that
$$y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = x$$

2. Let
$$w = \cos xy + y\cos x,$$
where
$$x = e^{-t} + 3s,\ y = 5e^{2t} - \sqrt{s}$$
Find $\dfrac{\partial w}{\partial t}$ and $\dfrac{\partial w}{\partial s}$.

3. If
$$yz^4 + xz^3 = e^{xyz}$$
find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

4. Let
$$f(x, y, z) = x^2y + x\sqrt{1+z}$$
   (a) Find $\nabla f(1, 2, 3)$, the gradient of the function at $(1, 2, 3)$.
   (b) Find $D_v f(1, 2, 3)$, the directional derivative of f at $(1, 2, 3)$ in the direction of $\vec{v} = 2\vec{i} + \vec{j} - 2\vec{k}$.

5. (a) Find the direction in which $f(x, y, z) = ze^{xy}$ increases most rapidly at the point $(0, 1, 2)$.
   (b) What is the maximum rate of increase?
   (c) What is the largest rate of decrease of f at this point? In which direction does this change occur?
   (d) When is the directional derivative at this point is half of its maximum value?

6. (a) Find a unit normal vector to the surface $\cos(xy) = e^z - 2$ at $(1, \pi, 0)$.
   (b) Write an equation of the tangent plane to the surface at $(1, \pi, 0)$.

7. Find the local maximum and local minimum values, and saddle points if any, of the function $f(x, y) = x^2 - y^2 + xy$

8. Find the local maximum and local minimum values, and saddle points if any, of the function $f(x, y) = e^{x+2y}(x^2 - y^2)$

9. Find the absolute maximum and minimum values of $f(x, y) = 3x^2y - x^3 - y^4$ on the closed triangular region in the xy-plane with the vertices $(0, 0)$, $(1, 1)$, and $(1, 0)$.