Math 251. WEEK in REVIEW 7. Fall 2013

1. Sketch the curve $r^2 = \cos 2\theta$. Find the area inside the curve.

2. Use a double integral in polar coordinates to evaluate the area of the region inside the circle $r = 4\sin\theta$ and outside the circle $r = 2$.

3. Use polar coordinates to evaluate

$$\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \ln(x^2+y^2+1)\,dxdy$$

4. Find the volume of the solid bounded by the surfaces

$$z=\sqrt{64-x^2-y^2} \text{ and } z=\frac{1}{12}(x^2+y^2)$$

5. (a) Find the mass of the plate bounded by $x^2 + y^2 = 4$, $x^2 + y^2 = 9$, if $x \leq 0$ and $y \geq 0$ if the density is $\delta(x, y) = \frac{y-4x}{x^2+y^2}$.

(b) Find the center of mass of the plate bounded by

$$y = x^2 \text{ and } y = x$$

if the density at any point is proportional to the distance from the y-axis.

6. Find the volume of the region bounded by the parabolic cylinders $z = x^2$, $z = 2 - x^2$ and by the planes $y = 0$, $y + z = 4$.

7. Write the equation $x^2 + y^2 + z^2 = 4y$ in cylindrical and spherical coordinates.

8. Convert the integral

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \int_{x^2+y^2}^{\sqrt{x^2+y^2}} xyz\, dz\, dx\, dy$$

to an integral in cylindrical coordinates, but don't evaluate it.

9. Convert the integral

$$\int_0^3 \int_0^{\sqrt{9-y^2}} \int_0^{\sqrt{9-x^2-y^2}} (x^2 + y^2 + z^2)\, dz\, dx\, dy$$

to an integral in spherical coordinates, but don't evaluate it.

10. Use cylindrical and spherical coordinates to find the volume of the ice-cream cone bounded above by the sphere $x^2 + y^2 + z^2 = 5$ and below by the cone $z = 2\sqrt{x^2 + y^2}$.

11. Let E be the solid region bounded below by the cone $z^2 = x^2 + y^2$, $z \geq 0$ and above by the plane $z = 4$. Set up the integral for computing the mass of the solid if the density is $\rho(x, y, z) = xyz$ in

    (a) cylindrical coordinates

    (b) in spherical coordinates

12. Find the center of mass of a solid right circular cone (the centroid of the cone) with height H and base radius R assuming that the density is homogeneous.