Math 251. WEEK in REVIEW 9. Fall 2013

- 1. Given the vector field $\vec{F} = z\vec{i} + 2yz\vec{j} + (x + y^2)\vec{k}$.
 - (a) Find the divergence of the field.

(b) Find the curl of the field.

(c) Is the given field conservative? If it is, find a potential function.

(d) Compute $\int_C z\,dx+2yz\,dy+(x+y^2)\,dz$ where C is the positively oriented curve $y^2+z^2=4, x=5.$

(e) Compute $\int_C z\,dx+2yz\,dy+(x+y^2)\,dz$ where C consists of the three line segments: from (0,0,0) to (4,0,0), from (4,0,0) to (2,3,1), and from (2,3,1) to (1,1,1).

2. Is there a vector field such that $\operatorname{curl} \vec{F} = -2x\vec{\imath} + 3yz\vec{\jmath} - xz^2\vec{k}$?

3. Identify the surface $x = \sqrt{u}\cos v, y = 5u, z = \sqrt{u}\sin v$.

4. Identify the surface which is the graph of the vector-function $\vec{r}(u,v) = \langle u+v, u-v, u \rangle$.

5. Find a parametric representation of the following surfaces:

(a)
$$x + 2y + 3z = 0$$
;

(b) the portion of the plane x + 2y + 3z = 0 in the first octant;

(c) the portion of the plane x + 2y + 3z = 0 inside the cylinder $x^2 + y^2 = 9$;

(d)
$$z + zx^2 - y = 0$$
;

(e)
$$y = x^2$$
;

(f) the portion of the cylinder $x^2+z^2=25$ that extends between the planes y=-1 and y=3

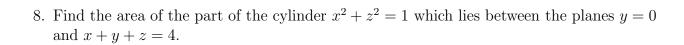
6. Find an equation of the plane tangent to the surface $x=u,y=2v,z=u^2+v^2$ at the point (1,4,5).

- 7. Given a sphere of radius 2 centered at the origin, find an equation for the plane tangent to it at the point $(1, 1, \sqrt{2})$ considering the sphere as
 - (a) the graph of $g(x,y) = \sqrt{4-x^2-y^2}$ (Note that the graph of g(x,y) is the upper half-sphere);

(b) a level surface of $f(x, y, z) = x^{2} + y^{2} + z^{2}$;

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(c) a surface parametrized by the spherical coordinates.



9. Find the area of the portion of the cone $x^2 = y^2 + z^2$ between the planes x = 0 and x = 2.