Math 251. WEEK in REVIEW 9. Fall 2013

1. Given the vector field $\vec{F} = z\vec{\imath} + 2yz\vec{\jmath} + (x + y^2)\vec{k}$.

   (a) Find the divergence of the field.

   (b) Find the curl of the field.

   (c) Is the given field conservative? If it is, find a potential function.

(d) Compute $\int_C z\,dx + 2yz\,dy + (x + y^2)\,dz$ where C is the positively oriented curve $y^2 + z^2 = 4, x = 5$.

(e) Compute $\int_C z\,dx + 2yz\,dy + (x + y^2)\,dz$ where C consists of the three line segments: from $(0, 0, 0)$ to $(4, 0, 0)$, from $(4, 0, 0)$ to $(2, 3, 1)$, and from $(2, 3, 1)$ to $(1, 1, 1)$.

2. Is there a vector field such that $\text{curl}\vec{F} = -2x\vec{\imath} + 3yz\vec{\jmath} - xz^2\vec{k}$?

3. Identify the surface $x = \sqrt{u}\cos v, y = 5u, z = \sqrt{u}\sin v$.

4. Identify the surface which is the graph of the vector-function $\vec{r}(u,v) =< u+v, u-v, u >$.

5. Find a parametric representation of the following surfaces:

(a) $x + 2y + 3z = 0$;

(b) the portion of the plane $x + 2y + 3z = 0$ in the first octant;

(c) the portion of the plane $x + 2y + 3z = 0$ inside the cylinder $x^2 + y^2 = 9$;

(d) $z + zx^2 - y = 0$;

(e) $y = x^2$;

(f) the portion of the cylinder $x^2 + z^2 = 25$ that extends between the planes $y = -1$ and $y = 3$

6. Find an equation of the plane tangent to the surface $x = u, y = 2v, z = u^2 + v^2$ at the point $(1, 4, 5)$.

7. Given a sphere of radius 2 centered at the origin, find an equation for the plane tangent to it at the point $(1, 1, \sqrt{2})$ considering the sphere as

   (a) the graph of $g(x, y) = \sqrt{4 - x^2 - y^2}$ (Note that the graph of $g(x, y)$ is the upper half-sphere);

   (b) a level surface of $f(x, y, z) = x^2 + y^2 + z^2$;

(c) a surface parametrized by the spherical coordinates.

8. Find the area of the part of the cylinder $x^2 + z^2 = 1$ which lies between the planes $y = 0$ and $x + y + z = 4$.

9. Find the area of the portion of the cone $x^2 = y^2 + z^2$ between the planes $x = 0$ and $x = 2$.