MATH 251, Section ______ Quiz#1 (Sections 11.1, 11.2, 11.3)

Thursday, Sept. 2, 2010 Dr. M. Vorobets

Due Tuesday, Sept. 7, 2010

NAME (print): ______________________________

No credit for unsupported answers will be given. Clearly indicate your final answer

1. Find the direction angles of the vector $\vec{a} = -2\vec{\imath} + 3\vec{\jmath} + \vec{k}$.

   SOLUTION. The magnitute of the vector $\vec{a}$ is

$$|\vec{a}| = \sqrt{(-2)^2 + (3)^2 + (1)^2} = \sqrt{14}$$

   The direction cosines are

$$\cos\alpha = \frac{a_1}{|\vec{a}|} = \frac{-2}{\sqrt{14}}, \quad \cos\beta = \frac{a_2}{|\vec{a}|} = \frac{3}{\sqrt{14}}, \quad \cos\gamma = \frac{a_3}{|\vec{a}|} = \frac{1}{\sqrt{14}}$$

   The direction angles are

$$\alpha = \arccos\left(\frac{-2}{\sqrt{14}}\right) = \pi - \arccos\frac{2}{\sqrt{14}} \approx 122.3^o, \quad \beta = \arccos\frac{3}{\sqrt{14}} \approx 36.7^o,$$

$$\gamma = \arccos\frac{1}{\sqrt{14}} \approx 74.5^o$$

2. Find the scalar projection and vector projection of $\vec{b} = 2\vec{\imath} + 3\vec{\jmath} - \vec{k}$ onto $\vec{a} = \vec{\jmath} - 2\vec{k}$.

   SOLUTION. The scalar projection of of $\vec{b}$ onto $\vec{a}$ is

$$\text{comp}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|} = \frac{(2)(0) + (3)(1) + (-1)(-2)}{\sqrt{(1)^2 + (-2)^2}} = \frac{5}{\sqrt{5}} = \sqrt{5}$$

   The vector projection of $\vec{b}$ onto $\vec{a}$ is

$$\text{proj}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\vec{a} = \frac{5}{5} < 0, 1, -2 > = < 0, 1, -2 >$$

3. Find two vectors orthogonal to both $< 2, 1, -1 >$ and $< 0, 1, 1 >$.

   SOLUTION. Let $\vec{a} = < 2, 1, -1 >$ and $\vec{b} = < 0, 1, 1 >$. Vectors $c = \vec{a} \times \vec{b}$ and $-\vec{c}$ are orthogonal to both $\vec{a}$ and $\vec{b}$.

$$\vec{c} = \vec{a}\times\vec{b} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 2 & 1 & -1 \\ 0 & 1 & 1 \end{vmatrix} = \vec{\imath}\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} - \vec{\jmath}\begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix} + \vec{k}\begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} = 2\vec{\imath} - 2\vec{\jmath} + 2\vec{k}$$

   The vector orthogonal to both $\vec{a}$ and $\vec{b}$ is

$$\vec{c}_1 = < 2, -2, 2 >$$

   The second unit vector orthogonal to both $\vec{a}$ and $\vec{b}$ is

$$\vec{c}_2 = -\vec{c}_1 = < -2, 2, -2 >$$

4. Find the volume of the parallelepiped determined by the vectors $< 1, -1, 1 >$, $< 2, 0, -1 >$, and $< 0, -1, 3 >$.

SOLUTION. Let $\vec{a} =< 1, -1, 1 >$, $\vec{b} =< 2, 0, -1 >$, and $\vec{c} =< 0, -1, 3 >$. Then the volume of the parallelopiped determined by $\vec{a}$, $\vec{b}$, and $\vec{c}$ is $V = |\vec{a} \cdot (\vec{b} \times \vec{c})|$.

$$\vec{b} \times \vec{c}) = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 0 & -1 \\ 0 & -1 & 3 \end{vmatrix} = 1 \begin{vmatrix} 0 & -1 \\ -1 & 3 \end{vmatrix} - (-1) \begin{vmatrix} 2 & 1 \\ 0 & 3 \end{vmatrix} + 1 \begin{vmatrix} 2 & 0 \\ 0 & -1 \end{vmatrix} = -1 + 6 - 2 = 3$$

Thus $V = 3$.