MATH 251, Section ______ Quiz 2 (Section 11.4)
Thursday, Sept. 9, 2010 Dr. M. Vorobets

NAME (print): ______________________________

No credit for unsupported answers will be given. No calculators. Clearly indicate your final answer

1. *[3 pts.]*

   (a) Find **symmetric** equations for the line passing through the points $A(-1, 3, 2)$ and $B(2, 3, -1)$.

   SOLUTION. The vector parallel to the line is $\overrightarrow{AB} =< 3, 0, -3 >$. Thus, the symmetric equations for the line are

   $$\frac{x+1}{3} = \frac{z-2}{-3}, \quad y = 3$$

   or

   $$\frac{x-2}{3} = \frac{z+1}{-3}, \quad y = 3$$

   (b) Find the equation of the plane passing through the point $P(-1, 0, 4)$ and parallel to the plane $x + 2y + 5z = 3$.

   SOLUTION. The vector orthogonal to the second plane is $\vec{n} =< 1, 2, 5 >$. Since the planes are parallel, they will have the same normal vector. Thus, the equation of the plane is

   $$1(x+1) + 2(y-0) + 5(z-4) = 0 \text{ or } x + 2y + 5z - 19 = 0$$

2. *[3 pts.]* Find the intersection of the lines

   $L_1: \quad x = 1 + t, \quad y = 2 - t, \quad z = 3t$

   $L_2: \quad x = 2 - s, \quad y = 1 + 2s, \quad z = 3 + s$

   SOLUTION. For the lines to intersect we must be able to find one value for t and one value for s satisfying the following three equations:

   $$1 + t = 2 - s$$
   $$2 - t = 1 + 2s$$
   $$3t = 3 + s$$

   Solving the first two equations gives $s = 0$, and $t = 1$, and, checking, we see, that these values satisfy the third equation. Thus, the lines intersect when $t = 1$ and $s = 0$, or at the point $(2, 1, 3)$.

3. *[4 pts.]* Find **parametric** equations of the line of intersection of the planes $x + y - z = 0$ and $2x - 5y - z = 1$.

   SOLUTION. The direction vector for the line of intersection is $\vec{v} = \vec{n_1} \times \vec{n_1}$, where $\vec{n_1} =< 1, 1, -1 >$ is the normal vector for the first plane and $\vec{n_2} =< 2, -5, -1 >$ is the normal vector for the second plane.

$\vec{v} = \vec{n_1} \times \vec{n_2} = \vec{\imath}(-1-5) - \vec{\jmath}(-1+2) + \vec{k}(-5-2) = -6\vec{\imath} - \vec{\jmath} - 7\vec{k}.$

To find a point on the line of intersection, set one of the variables equal to a constant, say $y = 0$. Then the equations of the planes reduce to $x - z = 0$ and $2x - z = 1$. Solving this two equations gives $x = z = 1$. So a point on a line of intersection is $(1, 0, 1)$. The parametric equations for the line are

$$
\begin{aligned}
x &= 1 - 6t \\
y &= -t \\
z &= 1 - 7t
\end{aligned}
$$