MATH 251, Section ______ Quiz 4 (Sections 11.7, 12.1, 12.2).
Thursday, Sept. 23, 2010 Dr. M. Vorobets

NAME (print): ______________________________

No credit for unsupported answers will be given. No calculators. Clearly indicate your final answer

1. *[3 pts.]* Find the equation of the normal plane for the curve $\vec{r}(t) =< 2\sin(3t), t, 2\cos(3t) >$ at the point $(0, \pi, -2)$.

   SOLUTION. The normal plane at $(0, \pi, -2)$ has normal vector $\vec{r'}(\pi)$.

   $$\vec{r'}(t) =< 6\cos(3t), 1, -6\sin)3t >$$

   $$\vec{r'}(\pi) =< -6, 1, 0 >$$

   An equation of the normal plane is

   $$-6(x-0) + 1(y-\pi) + 0(z+2) = 0$$

   or

   $$-6x + y - \pi = 0$$

2. *[2 pts.]* Find $\dfrac{\partial^2 f}{\partial x \partial y}$ if $f(x, y) = e^{xy}$.

   SOLUTION. $f_x(x, y) = e^{xy}(xy)'_x = ye^{xy}$

   $f_{xy}(x, y) = (f_x(x, y))'_y = (ye^{xy})'_y = e^{xy} + ye^{xy}(xy)'_y = e^{xy} + xye^{xy}$

3. *[5 pts.]* Find the curvature of the curve $\vec{r}(t) =< 1 + t, 1 - t, 3t^2 >$.

   SOLUTION.

   $$\kappa(t) = \frac{|\vec{r'}(t) \times \vec{r''}(t)|}{|\vec{r'}(t)|^3}$$

   $\vec{r'}(t) =< 1, -1, 6t >$, $|\vec{r'}(t)| = \sqrt{1 + 1 + 36t^2} = \sqrt{2 + 36t^2}$

   $\vec{r''}(t) =< 0, 0, 6 >$

   $$\vec{r'}(t) \times \vec{r''}(t) = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 1 & -1 & 6t \\ 0 & 0 & 6 \end{vmatrix} = \vec{\imath}(-6) - \vec{\jmath}(6) + \vec{k}(0) =< -6, -6, 0 >$$

   $|\vec{r'}(t) \times \vec{r''}(t)| = \sqrt{36 + 36} = \sqrt{72}$

   Thus,

   $$\kappa(t) = \frac{\sqrt{72}}{(2 + 36t^2)^{3/2}}$$