MATH 251, Section ______ Quiz 5 (Sections 12.4, 12.5, 12.6).
Thursday, Sept. 30, 2010 Dr. M. Vorobets

NAME (print): ______________________________

No credit for unsupported answers will be given. No calculators. Clearly indicate your final answer

1. *[2 pts.]* Find the differential of the function $z = \ln(2x - 3y)$.

   SOLUTION. $dz = \dfrac{\partial z}{\partial x}dx + \dfrac{\partial z}{\partial y}dy$

   $\dfrac{\partial z}{\partial x} = \dfrac{2}{2x-3y}$, $\dfrac{\partial z}{\partial y} = \dfrac{-3}{2x-3y}$, thus

   $$dz = \frac{2}{2x-3y}dx - \frac{3}{2x-3y}dy$$

2. *[3 pts.]* Find dw/dt if $w = \dfrac{x}{y} + yz$, $x = \sqrt{t}$, $y = \cos(2t)$, and $z = e^{-3t}$.

   SOLUTION.
   $$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$$

   $\dfrac{\partial w}{\partial x} = \dfrac{1}{y}$, $\dfrac{\partial w}{\partial y} = -\dfrac{x}{y^2} + z$, $\dfrac{\partial w}{\partial z} = y$.

   $\dfrac{dx}{dt} = \dfrac{1}{2\sqrt{t}}$, $\dfrac{dy}{dt} = -2\sin(2t)$, $\dfrac{dz}{dt} = -3e^{-3t}$, so

   $$\frac{dw}{dt} = \frac{1}{2y\sqrt{t}} + \left(-\frac{x}{y^2} + z\right)(-2)\sin(2t) + y(-3)e^{-3t}$$

3. *[2 pts.]* Find the direction derivative of the function $f(x, y) = x^3 - 4x^2y + y^2$ in the direction of the vector $\vec{u} = \left\langle \dfrac{3}{5}, \dfrac{4}{5} \right\rangle$.

   SOLUTION. $D_{\vec{u}}f(x, y) = \nabla f(x, y) \cdot \vec{u}$.

   $\nabla f(x, y) =< f_x(x, y), f_y(x, y) >=< 3x^2 - 8xy, -4x^2 + 2y >$, so

   $$D_{\vec{u}}f(x, y) =< 3x^2 - 8xy, -4x^2 + 2y > \cdot \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle = \frac{3}{5}(3x^2 - 8xy) + \frac{4}{5}(-4x^2 + 2y)$$

4. *[3 pts.]* Find the equation of the tangent plane to the surface $x^2 + y^2 - z^2 - 2xy + 4xz = 4$ at the point (1,0,1).

   SOLUTION. An equation of the tangent plane to the surface $F(x, y, z) = 0$ at the point (x_0, y_0, z_0) is

   $$F_x(x_0, y_0, z_0)(x - x_0) + F_y(x_0, y_0, z_0)(y - y_0) + F_z(x_0, y_0, z_0)(z - z_0) = 0$$

$F(x, y, z) = x^2 + y^2 - z^2 - 2xy + 4xz - 4,$
$F_x(x, y, z) = 2x - 2y + 4z,\ F_x(1, 0, 1) = 6,$
$F_y(x, y, z) = 2y - 2x,\ F_y(1, 0, 1) = -2,$
$F_z(x, y, z) = -2z + 4x,\ F_z(1, 0, 1) = 2.$
Thus, an equation of the tangent plane is

$$6(x-1) - 2(y-0) + 2(z-1) = 0$$

or

$$6x - 2y + 2z - 8 = 0$$