MATH 251, Section ______ Quiz 6 (Sections 12.7, 12.8).
Thursday, Sept. 30, 2010 Dr. M. Vorobets

The quiz is **due Tuesday, Oct. 5 at the beginning of class.**

NAME (print): ________________________________

No credit for unsupported answers will be given. Clearly indicate your final answer

1. *[3 pts.]* Find the local minimum and maximum, if any, of the function $f(x, y) = x^2 + xy + y^2 - 2x - y$.

   SOLUTION. We first locate the critical points:

   $f_x(x, y) = 2x + y - 2$

   $f_y(x, y) = x + 2y - 1$

   Setting these derivatives equal to zero, we get the following system:

   $$\begin{cases} 2x + y - 2 = 0 \\ x + 2y - 1 = 0 \end{cases}$$

   Substitute $y = 2 - 2x$ from the first equation into the second equation:

   $x + 2(2 - 2x) - 1 = 0$

   $x + 4 - 4x - 1 = 0$

   $-3x + 3 = 0$

   $x = 1$

   $y = 2 - 2x = 2 - (2)(1) = 0$

   The critical point is $(1, 0)$.

   Next we calculate the second partial derivatives:

   $f_{xx}(x, y) = 2$, $f_{xy}(x, y) = 1$, $f_{yy}(x, y) = 2$.

   Then

   $$D(x, y) = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 4 - 1 = 3 > 0$$

   Since $f_{xx}(x, y) = 2 > 0$, the function has a local minimum at the point $(1, 0)$, $f(1, 0) = -1$.

2. *[3 pts.]* Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = 6 - 4x - 3y$ subject to the constraint $x^2 + y^2 = 1$.

   SOLUTION. Let $g(x, y) = x^2 + y^2$.

   $\nabla f(x, y) =< -4, -3 >$, $\nabla g(x, y) =< 2x, 2y >$.

   Using Lagrange multipliers, we solve the equations

   $$\begin{aligned} -4 &= \lambda(2x) \\ -3 &= \lambda(2y) \\ x^2 + y^2 &= 1 \end{aligned}$$

From the first equation $x = -\dfrac{2}{\lambda}$, from the second equation $y = -\dfrac{3}{2\lambda}$ Plugging the expressions for x and y into the third equation, gives

$$\frac{4}{\lambda^2} + \frac{9}{4\lambda^2} = 1$$

or $\lambda^2 = \dfrac{25}{4}$, so $\lambda = 5/2$ or $\lambda = -5/2$

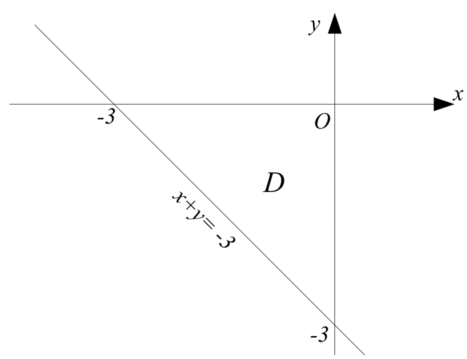
When $\lambda = \dfrac{5}{2}$, then $x = -\dfrac{4}{5}$, $y = -\dfrac{3}{5}$, and $f(-4/5, -3/5) = 11$

When $\lambda = -\dfrac{5}{2}$, then $x = \dfrac{4}{5}$, $y = \dfrac{3}{5}$, and $f(4/5, 3/5) = 1$

Thus, the minimum value is $f(4/5, 3/5) = 1$ and the maximum value $f(-4/5, -3/5) = 11$

3. *[4 pts.]* Find the absolute maximum and minimum values of the function $f(x, y) = x^2 + y^2 - xy + x + y$ if $D = \{(x, y) \in \mathbb{R}^2 | x \leq 0, y \leq 0, x + y \geq -3\}$.

SOLUTION. The set D is bounded by lines $x = 0$, $y = 0$, and $x + y = -3$



First we find critical points for f:

$f_x(x, y) = 2x - y + 1 = 0$

$f_y(x, y) = 2y - x + 1 = 0$

so the only critical points is $(-1, -1)$.

$f(-1, -1) = -1$

Now we look at the values of f on the boundary of D.

If $x = 0$, then $f(0, y) = y^2 + y$

$f_y(0, y) = 2y + 1 = 0$, so $y = -1/2$.

$f(0, -1/2) = -1/2$

If $y = 0$, then $f(x, 0) = x^2 + x$, $f_x(x, 0) = 2x + 1 = 0$ and $x = -1/2$

$f(-1/2, 0) = -1/2$

If $x + y = -3$, then $y = -3 - x$, and $g(x) = f(x, y) = x^2 + y^2 - xy + x + y = x^2 + (-3 - x)^2 - x(-3 - x) + x + (-x - 3) = 3x^2 + 9x + 6$

$g'(x) = 6x + 9 = 0$, so $x = -3/2$ and $y = -3 - (-3/2) = -3/2$

$f(-3/2, -3/2) = -3/4$

Finally we have to find the value of f at the points $(-3, 0)$ and $(0, -3)$:

$f(-3, 0) = 6$

$f(0, -3) = 6$

Thus, the absolute maximum value of the function is $f(-3, 0) = f(0, -3) = 6$ and the absolute minimum value is $f(-1, -1) = -1$.